GR8677 Solutions

Detailed Solutions to the GRE Physics Exam

http://grephysics.yosunism.com

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Problem 1 _______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=1*
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Newtonian*

This problem can be translated into an equation:

$$m\ddot{x} = -k\dot{x} - mg. \tag{1}$$

Note that the sign is compensated for. $\dot{x}$ is positive going up (so that $-k\dot{x}$ acts in the same direction as gravity), and it is negative going down (acting in the opposite direction of gravity).

(A) $\ddot{x} \neq g \forall t$ because $\ddot{x} = \frac{1}{m}(-k\dot{x} - mg)$

(B) True. $\dot{x} = 0$ at the top. Thus, plug that into the equation in described in (A) and get $\ddot{x} = -g$

(C) $\ddot{x} = \frac{1}{m}(-k\dot{x} - mg)$ could be greater than g, especially when $\dot{x}$ is positive in sign.

(D) The wind friction $(-k\dot{x})$ term would change sign when going downwards. (See the Note above.) Thus, the equation of motion presented above would be different in sign going down and up. (Also, one could argue that after the force of wind friction acts, to conserve energy, kinetic energy must lessen in the final energy sum. Thus, the final velocity can't be the same as v_0. Or, intuitively, one could see friction as a symmetry breaker, viz., the ideal parabolic path is disturbed in such a way that the end velocity changed.)

(E) Since the force other than gravity is a frictional force, which, by definition, slows down the object, the final velocity has to be less than v_0.

Problem 2 _______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=2*
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Vector*

While there may be a more quantitative solution, the simplest solution is qualitative, based on elementary vector addition and knowledge of the force center.

The problem states that the object orbits the Earth in a perfect circle, initially. This means that the initial velocity $(\vec{v_0})$ is perpendicular to the vector pointing to the earth center $(\vec{n})$, i.e., it's tangent to the circular path. This is the condition for uniform circular motion (the centripetal acceleration is due to the Gravitational Law).

After firing a missile straight to Earth center, its velocity gains an extra normal component $(-\vec{v_n})$, equal and opposite to the velocity of the missile fired to Earth. Thus, its trajectory would deviate from the circular trajectory.

Because the only source of acceleration comes from the Earth center, $-\vec{v_n}$, which is parallel to the centripetal acceleration provided by the Earth, will eventually go to 0. Recall that acceleration does not effect velocity components in the perpendicular direction (to wit: a projectile fired on Earth has the same constant v_x, but its v_y changes). There will thus always be a (nearly constant) tangential velocity, even at the perigees. However, $-\vec{v_n}$ will go to 0 at the perigees. The tangential velocity will remain more-or-less constant, so that instead of spiraling inwards, the path becomes an ellipse, as $-\vec{v_n}$ is restored at the apogees and zero'ed at the perigees.

(In a more down-to-earth form, this problem is essentially a projectile firing question with no numerical work involved.)

Problem 3 _______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=3*
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Wave Equation*

Simply and elegantly stated: $\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial x^2}$. (One can check this by noting that the dimensions cancel out to $meters^{-2}$ on both sides.) Now, $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$. Elsewhere than a vacuum, it's $v = \frac{1}{\sqrt{\epsilon\mu}}$, where, in this problem, we have $\epsilon = 2.1\epsilon_0$ and $\mu = \mu_0$.

No need to narrow the choices down. (D) is right because $v = \frac{1}{\sqrt{2.1 \times 1 \epsilon_0 \mu_0}} = \frac{c}{\sqrt{2.1}}$, where the last equality comes from substituting the definition of the speed of light (via an epsilon and a mu).

Problem 4 ________________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=4*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Wave Phenomena → *Wave Equation*

Perhaps this formula is more familiar: $y = A\sin(\omega t - kx)$. But then again, they define the familiar quantities k and ω for you in theirs...

(A) The amplitude is A. (Recall that the amplitude of $y = sin(x)$ is just 1, not 2.)

(B) How exactly does the argument of sin make it a traveling wave? Well, a traveling wave keeps the same waveform at all times. Thus, the overall argument has to be constant. $\omega t - kx = constant \Rightarrow x \propto -constant + \omega t$. This looks like the old high school kinematics equation $x = -constant + vt$. In fact, it's basically the same thing. That high school equation describes a particle going in the *positive x-direction*. So does this equation.

(C) Dimensions don't match. Period has units of time, not units of time/meter.

(D) The speed of the wave is $v = \lambda/T$. From the kinematics explanation explained in (B), the velocity of the wave is obviously $v = \frac{\omega}{k} = \frac{2\pi\lambda}{2\pi T}$

(E) True. See (D) and (B).

Problem 5 ________________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=5*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation*

This is a three step problem involving conservation of energy in steps one and three and conservation of momentum in step two.

1. $Mgh_0 = \frac{1}{2}Mv_1^2$... Conservation of energy determines the velocity of putty A, v_1, (of mass M) at the bottom of its trajectory, *right before* it intersects B.

2. $Mv_1 = (M + 3M)v_2 = 4Mv_2$... Since the putty thingies stick together, conservation of momentum is easy. Solve for v_2.

3. $\frac{1}{2}4Mv_2^2 = 4Mgh_3$... Conservation of energy, again. Solve for h_3.

And voila, plug in the relevant quantities to find that the answer is (A) $\frac{1}{16}h_0$.

Problem 6 ________________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=6*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Vectors*

Since there is only one force acting, i.e., the gravitational force, one can find the tangential acceleration by projecting $\vec{g}$ in the tangential direction. Equivalently, one dots gravity with the tangential unit vector, $\vec{v_t} = \vec{g} \cdot \hat{t}$.

There's a long way to do this, wherein one writes out the full Gibbsean vector formalism, and then there's a short and elegant way. (The elegant solution is due to Teodora Popa.)

The problem gives $f(x) = y = x^2/4$. Thus, $df/dx = dy/dx = x/2 = \tan\alpha$, where in the last step, one notes that the ratio dy/dx forms the tangent of the indicated angle.

One recalls the Pythagorean identity $\sin^2\alpha + \cos^2\alpha = 1$, and the definition of $\tan\alpha$ in terms of $\sin\alpha$ and $\cos\alpha$. Thus, one gets $x/2 = \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}}$. Square both sides to get $x^2/4 = \tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\sin^2\alpha}{1-\sin^2\alpha}$.

Solve $x^2/4 = \frac{\sin^2\alpha}{1-\sin^2\alpha}$ to get $\sin^2\alpha = \frac{x^2/4}{1+x^2/4} = \frac{x^2}{4+x^2}$.

The angle between the vectors $\vec{g}$ and $\hat{t}$ is $\pi/2 - \alpha$, and thus the tangential acceleration is $g \sin\alpha = \frac{gx}{\sqrt{x^2+4}}$.

Beautiful problem.

Problem 7 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=7*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Statics*

Straight-forward Newtonian statics:

$$F_x = 0 = F - T\sin(\theta) \Rightarrow T\sin(\theta) = F \tag{2}$$

$$F_y = 0 = T\cos(\theta) - mg \Rightarrow T\cos(\theta) = mg \tag{3}$$

Divide the two equations above, cancel T's, and get:
$\tan(\theta) = \frac{F}{mg} = \frac{10}{2*10} = .5$. Choice A is right.

Problem 8 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=8*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation*

Straight-forward conservation of energy problem:

$$E_0 = \frac{1}{2}mv^2 = E_f = Fl, \tag{4}$$

where $l = 0.025m = \frac{25}{1000}$, $m = 5kg$, and $v = 10m/2$. Plug everything in, and solve for F, to get choice D, $10,000N$.

Problem 9 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=9*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Current Density*

There are (at least) two ways to solve this problem. If one has a decent memory or might make for a decent experimenter, one should remember that the electron drift velocity is on the order of $10^{-4}m/s$. Choice D is the only thing that fits this order. If one is a theorist who can't remember jack, then one would have to do a bit more work:

Recall the basic equation:

$$j = \frac{I}{A} = nev, \tag{5}$$

where $n = \frac{N}{V} = 1E28$, i.e., the number of electron per volume, and v is the drift velocity. The area is $A = \pi(\frac{d}{2})^2$.

Solving for v, and making the appropriate approximate plug-in's:

$$\frac{4I}{\pi d^2}\frac{1}{ne} \approx \frac{4I}{\pi(\frac{2}{100})^2}\frac{1}{(1E28)(1.5E-19)} \approx 3E-4, \tag{6}$$

which is closest to choice D.

Problem 10 $\underline{\hspace{5cm}}$ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=10*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Gauss' Law*

Only choices C, D, and E make sense, since the field at $r = 0$ has to be 0. Moreover, since a uniform charge distribution is applied, the field has to increases linearly within the volume of the sphere. Choice D and E are both out, leaving just C.

Direct verification via applying Gauss' Law for the inside of the sphere: $E(4\pi r^2) = Q = \rho\frac{4}{3}\pi r^3 \Rightarrow E = \frac{\rho r}{3} \propto kr$.

Outside, all five choices converge to an inverse-square decreasing field—so, no worry there!

Problem 11 $\underline{\hspace{5cm}}$ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=11*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Vector Calculus*

There are two identities from vector calculus one has to know by heart. The one directly applicable to this problem is:

$$\nabla \cdot (\nabla \times \vec{H}) = 0 \tag{7}$$

Plug in the equation given in the problem to the identity above to get 0.

(The other identity, not quite as useful for this problem, but perhaps useful for subsequent problems, is: $\nabla \times (\nabla f)$)

Problem 12 $\underline{\hspace{5cm}}$ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=12*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Wave Phenomena → *Doppler Effect*

For a source that's coming towards you, its frequency will obviously be higher than its original source frequency. Choices A and B are out. To know for sure whether the choice is C, D, or E, one can apply the Doppler Effect.

The Doppler Effect equation can easily be derived even if one forgets it. A source f_0 approaching at v_0 would have $\Delta\lambda$, the distance between waves, *decreasing*. $v = \lambda f \Rightarrow \Delta\lambda = \frac{v_{sound} - v_0}{f_0}$. The $\Delta\lambda$ wave travels at the speed of sound, thus the frequency you receive is: $f = \frac{v_{sound}}{\Delta\lambda} = 10 f_0$. Choice E.

Problem 13 $\underline{\hspace{5cm}}$ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=13*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Interference*

Fact: The human eye can only see things in motion up to about 25 Hz. (One can approximate this knowing that the human eye blinks once every three seconds on average.)

Now, the problem mentions that the relative phase is varied, at a constant frequency of 500 Hz, which is much greater than the maximum frequency of the human eye. Interference is produced as long as the sources are coherent, and the sources are coherent as long as there's a constant relation between relative phase in time.

(A) The frequency of the phase change has nothing to do with the color of light.
(B) Interference pattern is different for π and 2π phase changes...
(C) Interference can exist for other phase differences.
(D) One can have interference even with polychromatic sources.

(E) The interference pattern shifts position (since the source remains coherent from the constant relation with relative phase to time) at a rate too fast for the human eye, as explained above.

Problem 14 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=14*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Exact differentials*

The key equation is: $PV = nRT$, and its players, P, V, n, R, T, are terms one should be able to guess.

(A) True, according to the ideal gas law. (This is also the final step in deriving Mayer's Equation, as shown below.)

(B) This translates into the statement $\left.\frac{dq}{dT}\right|_V = \left.\frac{dq}{dT}\right|_P \Rightarrow c_V = c_P$.The problem gives away the fact that for an ideal gas $C_P \neq C_V$. B can't be right.

(C) According to the ideal gas law, the volume might change.

(D) False. An ideal gas's internal energy is dependent only on temperature. More elegantly, $u = u(T)$.

(E) Heat needed for what?

If one is interested in the formal proof of the relation $c_p = c_v + nR$, read on about Mayer's equation:

For thermo, in general, there's an old slacker's pride line that goes like, "When in doubt, write a bunch of equations of states and mindlessly begin taking exact differentials. Without exerting much brainpower, one will quickly arrive at a brilliant result." Doing this,

$$Q = U + W \quad \Rightarrow \quad dQ = dU + PdV \tag{8}$$

$$U = U(T, V) \quad \Rightarrow \quad dU = \partial_T U|_T + \partial_V U|_V \tag{9}$$

$$PV = nRT \quad \Rightarrow \quad PdV + VdP = nRdT \tag{10}$$

Plugging in the first law of thermodynamics into the U equation of state, one gets $dQ = \partial_T U|_T + (\partial_T U|_V + P)\, dV = \partial_T U|_T + PdV$, where the last simplification is made by remembering the fact that the internal energy of an ideal gas depends only on temperature.

(Taking the derivative with respect to T at constant volume, one gets $\left.\frac{dQ}{dT}\right|_V = \partial_T U|_T = C_V$.)

Plugging in the simplified result for $dQ = ...$ into the third equation of state, the ideal gas equation, one gets: $dQ - C_v dT + VdP = nRdT$. Taking the derivative at constant pressure, one gets:

$$\left.\frac{dQ}{dT}\right|_P = C_V + nR \tag{11}$$

So, one sees that it is the ideal gas equation that makes the final difference. The work of an ideal gas changes when temperature is varied.

Problem 15 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=15*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Probability*

Probability is mostly common sense and adhering to definitions.

The probability that a gas molecule or atom is in the small cube is $P(in) = 1E-6$. The probability that it's not in that small cube is $P(not) = 1 - 1E-6$. Assuming independent gas molecules or atoms, i.e., the usual assumption of randomness in Stat Mech, one gets, $P(Ngasatoms) = (1 - 1E - 6)^N$.

The answer is thus C.

Problem 16 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=16*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Particle Physics*

The properties of a muon most closely resembles that of an electron. In the usual β-decay, an electron is emitted along with either a neutrino or an antineutrino. In a wildcard β-decay, a muon is emitted instead of an electron.

Problem 17 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=17*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Radioactivity*

$X_Z^A \rightarrow ? \rightarrow ? \rightarrow Y_{Z-1}^{A-4}$ *by natural decay*

Recall the following decay processes, and recall that the subscripts and superscripts have to balance on both sides of the arrow.

Beta (β^-) Decay:

$$X_Z^A \rightarrow X'^A_{Z+1} + \beta^0_{-1} + \bar{\nu}, \tag{12}$$

where $\bar{\nu}$ is the symbol of an anti-neutrino. (The β^+ decay is accompanied by a neutrino, ν, instead of an anti-neutrino.)

Alpha Decay:

$$X_Z^A \rightarrow X'^{A-4}_{Z-2} + He_2^4 \tag{13}$$

Gamma Decay (no change in A or Z):

$$X_Z^A \rightarrow X_Z^A \rightarrow +\gamma \tag{14}$$

Deuteron Decay (rare):

$$X_Z^A \rightarrow X_{Z-1}^{A-2} + H_1^2 \tag{15}$$

(A) $X_Z^A \rightarrow X_{Z+1}^A + \beta^0_{-1} \rightarrow X'^{A-4}_{Z-1} + He_2^4 \rightarrow Y_{Z-1}^{A-4}$. This works out.
(B) β^- decay *always* occurs with an antineutrino.
(C) γ emission doesn't change the atomic number.
(D) Although the numbers work out here, deuteron decay is rare, thus not natural.
(E) The subscripts and superscripts don't add up.

Problem 18 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=18*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Schrodinger Equation*

The problem gives the wave function, wherein the hidden mysteries of the problem are contained. The potential is referenced as $V(x)$, which means that it's time-independent. Thus, the Time-Independent Schrodinger Equation can be used: $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi + V(x)\psi = E\psi$. The second derivative of the given ψ is $\frac{d^2}{dx^2}\psi = (-b^2 + (b^2x)^2)\psi$. Plug that into the TISE, and one gets $-\frac{\hbar^2}{2m}(-b^2 + (b^2x)^2) = E - V(x)$

Now, plugging in the potential condition $V(0) = 0$, one gets, $\frac{\hbar^2}{2m}((b^2x)^2) = E - V(0) = E$. This implies that the term on the left that disappeared from that substitution is the $V(x)$ term. Therefore, one deduces that $V(x) = \frac{\hbar^2}{2m}(b^2x)^2$.

The right answer would be (B).

Problem 19 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=19*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Bohr Theory*

Recall the Rydberg energy. QED

Problem 20 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=20*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Rest Energy*

If one remembers the formulae from special relativity, arithmetic would be the hardest part of the problem.

The problem is to solve $\gamma m_k c^2 = m_p c^2$, where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ and $\beta = \frac{v}{c}$.

The rest-mass of both the kaon and the proton are given in the problem. Thus, the equation reduces to $\gamma = \frac{938}{494}$.

Now, because the range of velocities vary significantly between $1.x$ and 2, one can't directly approximate that as 2. Boo. So, long-division by hand yields approximately $1.9 = \gamma = \frac{1}{\sqrt{1-\beta^2}}$.

The author of this site prefers to use fractions instead of decimals. Thus $\gamma^2 = 1.9^2 = \left(1 + \frac{9}{10}\right)^2 = 1 + \frac{81}{100} + \frac{180}{100} = \frac{262}{100}$. Express γ in terms of β to get $1 - \beta^2 = \frac{100}{262} \approx \frac{1}{3} \Rightarrow \beta^2 = \frac{2}{3}$.

8×8 is about 64, so the velocity has to be greater than 0.8c. The only choice left is (E).

If one has the time, one might want to memorize the following:

$$\gamma(\beta = 0.1) = 1.005 \quad (16)$$
$$\gamma(\beta = 0.25) = 1.033 \quad (17)$$
$$\gamma(\beta = 0.5) = 1.155 \quad (18)$$
$$\gamma(\beta = 0.75) = 1.51 \quad (19)$$
$$\gamma(\beta = 0.9) = 2.29 \quad (20)$$

, or perhaps a more elaborate list of $\gamma - \beta$ correlations.

If one knew that before-hand, then one would immediately arrive at choice (E).

Problem 21 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=21*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Metric*

Recall that the metric $dS^2 = -dt^2 + dr^2$ (equivalently, $dS^2 = dt^2 - dr^2$)

$dr^2 = dx_i dx_i = (dx)^2 + (dy)^2 + (dz)^2 = 4 + 0 + 4 = 8$ and $dt^2 = 4$. Thus $dS^2 = 4$, and $S = 2$, as in choice (C).

Problem 22 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=22*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Lorentz Transformation*

When an electric field is Lorentz-transformed, afterwards, there might be both a magnetic and electric field (in transverse components). Or, more rigorously, one has,

$$\begin{aligned}
E'_x &= E_x & (21)\\
E'_y &= \gamma(E_y - vB_z) & (22)\\
E'_z &= \gamma(E_z + vB_y) & (23)\\
B'_x &= B_x & (24)\\
B'_y &= \gamma(B_y + E_z v/c^2) & (25)\\
B'_z &= \gamma(B_z - vE_y/c^2), & (26)
\end{aligned}$$

for motion in the x direction.

(A) Obviously not. Suppose initially, one has just E_x, afterwards, there's still E'_x.

(B) True, as can be seen from the equations above.

(C) Not true in general. Suppose $\vec{E} = E_x\hat{x} + E_y\hat{y} + E_z\hat{z}$. Afterwards, the transverse components are off by a γ even if there's no B field to start with.

(D) Nope.

(E) Mmmm... no need for gauge transformations.

Problem 23 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=23*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Solid State*

Actually, one can figure this one out with only knowledge of lower div baby physics.

(A) Electrical conductivities for conductors, semiconductors, and insulators go (in general), like this $k_c > k_s > k_i$. Thus, copper should be more conductive than silicon. This is true (but one is trying to find a false statement).

(B) The resistivity $\rho = \rho_0(1 + \alpha\Delta T)$, thus the conductivity, $\sigma = 1/\rho \propto 1/(\Delta T)$. As T increases, σ decreases. This is true.

(C) Silicon is a semi-conductor, and thus it probably does not follow the same relations as (B). In fact, semiconductors have *negative* α temperature coefficient of resistivities. Thus, $\rho < \rho_0$, which implies that $\sigma > \sigma_0$ for temperature increase.

(D) Doping a conductor like copper will just make it cheap. Think of cheap wire.

(E) Doping a semiconductor, however, can make it more conductive.

Problem 24 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=24*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Circuits*

Apply Kirchhoff's Loop Law: $(G - V) - I(R + 1) = 0$, where G is the voltage of the generator and V is the voltage of the battery, and the extra 1 is the internal resistance. $I = 10 Amperes$ in the problem, and thus $R = 1\Omega$.

Problem 25 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=25*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Lorentz Force*

Recall the Lorentz Force, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.

$\vec{E}$ and $\vec{B}$ are parallel. The particle is released from rest, so the Electric force would propel it. The resulting velocity would be parallel to the electric field, but since the magnetic field is also parallel to that, there would be no magnetic force contribution. The particle thus goes in a straight line.

Problem 26 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=26*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *X-Rays*

The K-series refers to the inner-most shell. (The order from inner to outer goes like K, L, M, N.) K-series refers to a transition from some outer state to the inner-most shell, where $n = 1$ in the usual Bohr equation,

$$E = -13.6(Z-1)^2\left(1 - 1/n_i^2\right) eV, \tag{27}$$

where $(Z-1)^2$ is used to account for shielding.

For electrons bombarding a target, one assumes that the electrons are coming from $n_i = \infty$, thus the equation becomes $E = -13.6(Z-1)^2 \approx 10 * 900 \approx 10,000$ eV, as in choice (D).

Problem 27 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=27*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Spin*

Spin explains a lot of things.

(A) Remember orbitals? Whether a shell is full or not determines the properties of each column of the periodic table. A full shell has all electron spins paired together, while a partially filled or empty shell doesn't have that. So, spin is definitely in the Periodic Table.

(B) The specific heat of metals differs if one calculates it using the Fermi-Dirac or Bose-Einstein distributions; the first is used for fermions and the second for bosons. So, spin plays a role here.

(C) The Zeeman effect has to do with splitting caused by spin.

(D) The deflection of a moving electron is due to the magnetic field contribution to the Lorentz Force. This is a classical non-spin related phenomenon, on first analysis. This is the best choice.

(E) Fine structure has to do with splitting caused by spin.

Problem 28 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=28*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Normalization*

Recall that $\int |\psi(x)|^2 dx = 1$ is the condition for a normalized function.

$$\begin{aligned} |\psi|^2 &= \int_0^{2\pi} |A|^2 d\phi && (28)\\ &= 2\pi|A|^2 && (29)\\ &= 1 && (30)\\ \Rightarrow A &= \frac{1}{\sqrt{2\pi}}, && (31) \end{aligned}$$

where the condition $|e^{im\phi}|^2 = e^{im\phi}e^{-im\phi} = 1$ is used.

This is choice (D).

Problem 29 — $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=29$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Right Hand Rule*

Note: this is a negative test charge. So, for the test charge, one has to either choose the opposite direction as that yielded by the Right Hand Rule or one could use the Left Hand Rule, which is just the RHR done with the left hand.

Suppose the current is a straight line pointing upwards along the page. The RHR for the current shows a magnetic field that's coming out of the page from the left side of the current and a field going into the page on the right side.

The problem wants the test charge to go parallel to the current. Applying the Lorentz Force, where $F \propto \vec{v} \times \vec{B}$, one finds that no matter the direction of approach, the only way for the force to point parallel to the current is for the velocity to go towards the wire. (Check this: Suppose the charge comes in from the left; the force would point parallel to the current. Suppose the charge comes in from the right; the force would point parallel to the current, again.)

Problem 30 — $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=30$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Orbitals*

Recall that:

$$
\begin{aligned}
s &= 0 && (32)\\
p &= 1 && (33)\\
d &= 2 && (34)\\
f &= 3 && (35)\\
g &= 4...(etc.), && (36)
\end{aligned}
$$

where the right side of the equal sign is the angular momentum l.

And then there's also the formula $2(2l + 1)$, which determines the number of electrons in each subshell.

(A) The configuration does not specify anything about $3d$. This is the first row of transitional metals.

(B) From the formula above, one sees that the s subshell needs 2 electrons to be filled. The $4s$ subshell is thus not filled.

(C) $l = 4$ corresponds to g. The configuration does not specify g.

(D) Because the problem states that potassium is in the ground state, the atomic number is the same as the number of electrons in the configuration. The sum of the superscripts is 19.

(E) The valance shell is $4s^2$. Many-electron atoms have wave functions that take after the hydrogen atom, and one recalls that ψ_{40m} looks like a spherically symmetrical blob of a ball. This is it.

Problem 31 — $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=31$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Photoelectric Effect*

Perhaps one recalls the "hot phrase" *stopping potential*. The $|eV|$ used in the photoelectric equation is essentially the stopping potential (energy). This memory-recall immediately narrows the choices down to just (A) and (C). Now, to find out if the potential is positive or negative.

FYI: The Photoelectric effect equation is basically just conservation of energy. One has $eV + W = hf$, where eV is the kinetic energy of the electron as it accelerates through the medium between the cathode and collector, W is the energy to free the electron from the metal, and hf is the energy given by the light source. Essentially, the energy from the photon first frees the electron from a sort of (valence) electron sea it lives in while in the metal, and then the excess energy propels it from cathode to collector in order to keep the current running. The minimum kinetic energy required to get the current going is the stopping potential (energy) eV. (According to the fight analogy below, the photon is the one who starts the fight, socking the electron off and away across the currents...)

The electron charge is negative and the potential V must be a negative quantity in order to make eV positive overall.

(On the lighter side... The Photoelectric effect is also related to the Compton Effect. The effects can be seen as an arena fight. Think WWWF, but with electrons and photons. In PEE, the photon knocks the electron out, while in CE, the electron retaliates! It knocks the photon off course.)

Problem 32 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=32*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Photoelectric Effect*

(A) The photoelectric effect was derived before formal quantum mechanics and that angular momentum mess came along. Moreover, electron orbits don't really apply to the valance electron sea.

(B) This is true, but it doesn't help derive the photoelectric effect.

(C) Nah, there's also something weird called electron-electron annihilation. Basically, two electrons crash into each other and a photon is created. (Perhaps some other particles, too.) Also, think of your regular desktop lamp—light is emitted, but the electrons are probably not jumping between orbits.

(D) Right. Einstein won the Nobel Prize about a hundred years ago via his proposal that photons have quantized energy $E = h\nu$.

(E) As a pure-ideal theory, the photoelectric effect depends on a single photon exciting a single electron. It favors the particle view of light. Choice (E) is out.

Problem 33 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=33*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Photoelectric Effect*

The quantity W is also known as the work function, and it's the minimum amount of work that has to be done to free the electron from the electron sea. If one forgets whether it's the electron or photon that's being moved, then here's a useful mnemonic. Remember that the photoelectric effect has a photon knocking out an electron and the Compton effect has an electron knocking a photon off course. (For the full fight-club analogy, See Prob 32)

Problem 34 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=34*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Potential Energy*

Recall the lovely relation,

$$F_x = -\frac{\partial U}{\partial x} \Leftrightarrow \vec{F} = -\nabla U. \tag{37}$$

$U = kx^4 \Rightarrow F = -4kx^3$

Problem 35 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=35*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Hamiltonian*

Recall the lovely relation,

$$H = T + V, \tag{38}$$

where T is the kinetic energy and V is the potential energy. (And, while one is at this, one should also recall that the Lagrangian is $L = T - V$.)

The potential energy is given to be kx^4. The kinetic energy $\frac{1}{2}mv^2$ can be written in the form of $\frac{p^2}{2m}$. Thus, choice (A) is the right answer.

Problem 36 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=36*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Lagrangian*

Recall Hamilton's Principle (of least action),

$$\int_{t_1}^{t_2} L dt, \tag{39}$$

where $L = T - V$ is the Lagrangian and T is the kinetic energy and V the potential energy. The potential energy is given in the problem. The choice is obviously (A).

Problem 37 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=37*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Statics*

Sum over each component. Note that the horizontal direction has a net force proportional to the centripetal acceleration $a = v^2/r = \omega^2 r$, where $v = \omega r$. Note that T is the tension.

$$\sum F_x = T\sin(\theta/2) = mv^2/r = m\omega^2 r \tag{40}$$

$$\sum F_y = T\cos(\theta/2) - mg = 0 = \tag{41}$$

Solve for T above to get,

$$\sum T\sin(\theta/2) = m\omega^2 r \tag{42}$$

$$\sum T\cos(\theta/2) = mg \tag{43}$$

Find the magnitude of T and use the Pythagorean theorem,

$$||T|| = \sqrt{T^2\cos^2(\theta/2) + T^2\sin^2(\theta/2)} = m\left(g^2 + \omega^4 r^2\right)^{1/2}, \tag{44}$$

and thus (E) is the right answer.

(The above should be fairly obvious, but if one is totally clueless, then one can eliminate choice (D) from noting units. The angular velocity has units of $1/s$ but g^2 has time units proportional to $1/s^4$.)

Problem 38 *http://grephysics.net/disp.php?yload=prob&serial=1&prob=38*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Logical Circuit*

(A) This is the OR gate. Triangle with fat-end on input side denotes OR.
(B) Triangle with fat-end closer to output side denotes AND. (Pointy tip points to each input.)
(C) A 2 bit-adder involves more operations than this...
(D) A flip-flop is a sort of sandal that flips and flops. It might also flip the floating point.
(E) A fan-out describes the maximum number of outputs a circuit can excrete. (Fan-in would be inputs.)

Problem 39 — $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=39$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Lab Methods → *Log-Log Graph*

The key phrase is *most accurately expressed.* Choice (E) works because it applies decently well to the following two key points, one of them given, and the other one right next to the given: ($\omega = 3E5, 1E2$) and ($1E6, 1E1$)

Try a power-law relation: $Gain = K\omega^n \Rightarrow 1E2 = K(3E5)^n$ and $1E1 = K(1E6)^n$. Divide the two equations to get $1E1 = (3E-1)^n Rightarrow 10 = (1/3)^n$. The closest answer would be $n = -2$, which is case (E).

(Can't be an exponential relation, since the Gain would decrease *exponentially* as ω increases. It's not decaying nearly as fast.)

Problem 40 — $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=40$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Radioactivity*

Radioactive counting rates (number of decay per unit time) follow the Poisson Distribution. (Recall that the Poisson Distribution describes the results of experiments in which one counts events that occur independently, thus at random, but at a definite average rate.) In the PD, $\sigma = \sqrt{\bar{x}}$, i.e., the standard deviation is approximately the square-root of the average.

Suppose $9934 \approx 10000$ is the average. The square root of that is 100, hence (A) is the answer.

(Depending on whether one is in the mood for rolling the celestial self-loaded 5-sided dice, one can look for the most obvious relationship between numbers in problems. $\sqrt{x}, x^2$ are two commonly used relationships.)

The Normal (Gaussian) distribution describes the distribution of values for any measurement subject to many sources of error that are all random and small. But, the Poisson Distribution fits this case more closely.

Problem 41 — $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=41$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Binding Energy*

Nickel and Iron are the most tightly bound nuclei, thus have the highest binding energy. Nickel isn't on the list, thus Iron must be the choice.

Problem 42 — $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=42$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Scattering Cross Section*

The target nuclei per cm^3 is $\rho = N/V = 1E20nuclei/cm^3$.

The scatterer thickness is $t = 1E - 1cm$.

Passing through the scatterer, only $P = 1E - 6$ particles are scattered. (This is the probability of scattering.)

One can obtain to formula for the scattering cross section from ordinary dimensional analysis spiced up with some common sense (or recalling its definition back in Marion and Thornton's *Dynamics* book). It is:

$$A = P\frac{V}{Nl} = \frac{P}{\rho l} = \frac{1E-6}{1E20 \times 1E-1} = 10^{-6-20+1} = 10^{-25}, \tag{45}$$

which is choice (C).

Problem 43 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=43*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Normal Modes*

Because there are two degrees of freedom in this problem, there are two normal mode frequencies.

Because there is no external torque acting on the system, the center of mass of the system stays the same through time.

From common sense, one deduces that ω_1 has to do with the outer masses moving perfectly out of phase, i.e., masses A and C moving either towards the left and right (away from each other), respectively, or right and left (towards each other), respectively—and B being perfectly stationary, thereby "conserving" the center of mass.

The other angular frequency, ω_2, has to do with either masses A and C moving in phase and mass B out of phase.

ω_1 is actually equivalent to having a single mass on a string, since because the middle mass doesn't move, it acts as a sort of support for the spring. $\omega_1 = \sqrt{k/m}$, which would correspond to choice (B). (Of course, one should recall the obvious, that $\omega = 2\pi f$.)

(Incidentally, one can derive ω_2 without having to resort to the formalism of matrix mechanics: Since the center of mass remains 0, one has $R = \frac{mx-2mX+mx}{m+2m+m}$. Solving, one gets $X = x$. The displacement of the middle mass, mass B, is thus $x + X = 2x$, while the displacements of the smaller masses, masses A and C, are both x. The displacement of each spring is $2x$. Potential energy is thus $U = 2\frac{1}{2}k(2x)^2 = \frac{1}{2}8kx^2$. The kinetic energy is $T = 2\frac{1}{2}m(\dot{x})^2 + \frac{1}{2}(2m)(\dot{x})^2 = \frac{1}{2}4m(\dot{x})^2$. The normal mode frequency is deduced by $\omega_2 = \sqrt{\frac{k_{eff}}{m_{eff}}} = \sqrt{\frac{8k}{4m}} = \sqrt{\frac{2k}{m}}$)

Problem 44 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=44*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Momentum*

One could use energy, but then one would have to take into account the inertia. Momentum might be easier,

$$(p_i = mv) = (p_f = MV) \Rightarrow V = \frac{m}{M}v, \tag{46}$$

where the final momentum takes into account the fact that the final velocity of the particle is at rest (0). And, so it is (A)!

Problem 45 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=45*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *de Broglie Wavelength*

This problem can be solved via the Compton Effect equation $\Delta\lambda = \lambda_c(1-\cos\theta)$, where θ is the angle of the scattered photon and $\lambda_c = h/(m_e c)$ is the so-called Compton Wavelength. In this case, since the particles are protons, one has $\lambda_c = h/(m_p c)$

Since the photon scatters off at 90 degrees, the equation simplifies to $\Delta\lambda = \lambda_c$. This is the increase in wavelength, as in choice (D).

To a certain degree, one can hand-wave this problem via the following method:

Recall the de Broglie relation,

$$p = \frac{h}{\lambda} \tag{47}$$

, where λ is the wavelength, p is the momentum and h is Planck's constant.

The momentum of the proton is $m_p c$, thus $\lambda = \frac{h}{m_p c}$.

Problem 46 — *http://grephysics.net/disp.php?yload=prob&serial=1&prob=46*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Blackbody Radiation*

Recall the T^4 law

$$\sigma \propto T^4, \tag{48}$$

where σ is the energy density.

Apply that law. $10 = T_1^4$ and $x = 2^4 T_1^4$, where mW is the units. Compare the two to get $x = 2^4 \times 10mW = 160mW$. This is choice (E).

Problem 47 — *http://grephysics.net/disp.php?yload=prob&serial=1&prob=47*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Franck-Hertz Experiment*

Like the typical experiment in QM, the Frank-Hertz experiment has to do with shooting a bunch of particles through some oven. Its significance is in early observations of energy levels. Also, its key conclusion is that electrons are scattered *elastically*.

But, if one doesn't know the above, one can always use MOE—the Method of Elimination.

(A) Elastic collision requires conservation of kinetic energy. A bit restraining, but keep the choice.

(B) Never scattered elastically? Never is too strong a word to be favored by ETS.

(C) Seems reasonable-ish. Keep it.

(D) This is, again, too restraining. It makes sense that depending on the impact angle and momentum, different amount of energy would be lost.

(E) Discrete energy lost is mentioned in (C), but again, this choice is much too restraining, stating that "there is no energy range..."—too strong of a phrase to be favored by ETS.

Problem 48 — *http://grephysics.net/disp.php?yload=prob&serial=1&prob=48*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Transition*

Transitions go from state one to another state that's completely unique relative to the first state. If both initial and final states have spherically symmetrical wave functions, then they have the same angular momentum. This is forbidden.

Problem 49 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=49*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Schrodinger Equation*

Recall the Time-independent Schrodinger Equation,

$$H\psi = E\psi \Rightarrow -\frac{\hbar^2}{2m}\psi'' + V(x)\psi = E\psi, \tag{49}$$

where $H = -\frac{\hbar^2}{2m}\psi'' + V(x)\psi = E\psi$.

The classical Hamiltonian is $H = \frac{p^2}{2m} + V(x)$, where one sees the only difference is $p \rightarrow \frac{\hbar}{i}\frac{d}{dx}$. From the classical Hami, one can directly reach the Hami operator in the TISE via substituting a differential operator for momentum, as in choice (B).

Problem 50 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=50*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Solid State Physics*

Recall $R_H = -\frac{1}{nec}$, where the sign shows that the charge carrier is negative. (n is the electron density, e electron charge, c speed of light)

The Hall effect has to do with a bar of metal placed in a magnetic field. An electric current runs through it perpendicular to the field direction. However, the magnetic field term in the Lorentz force creates a force perpendicular to both the magnetic field and electric field. This "pulls" the current astride from its original straight path. Charge starts accumulating on one side of the slab from this force, even as current continues to flow perpendicular to the accumulation direction. In equilibrium, the magnetic force balances the electric force from this charge accumulation. Apply some basic EM theory, and one can derive the Hall coefficient (R_H) defined above.

Problem 51 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=51*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Specific Heat*

Both Debye and Einstein assumed that there are 3N oscillators. (In fact, one can argue that the core of condensed matter begins with the assumption that a continuum piece of matter is basically a tiny mattress—a bunch of springs laden together.) Answer is thus (B).

However, Einstein was too lazy, and he decided that all 3N oscillators have the same frequency. Debye assigned a spectrum of frequencies (phonons).

Problem 52 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=52*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Potential*

By Gauss Law, if there are no charges inside the cube, then the electric field inside would be 0. The potential ϕ is related to the field $\vec{E}$ by $\vec{E} = \nabla\phi$, and thus since $\vec{E} = 0$, one infers that ϕ is constant. Since the potential function has to remain continuous its value everywhere inside of the cube is the same as that at the surface of the cube, which is given as V.

Problem 53 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=53*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Accelerating Charges*

One doesn't really need to understand the mess Griffiths has on accelerating charges to solve this problem. In fact, common sense works. Since the particle spends the most time about the origin, the field there should be maximal. The origin corresponds to $\theta = 90°$, via the system used in the diagram. This is choice (C).

Problem 54 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=54*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Dielectric*

Recall the following equations,

$$\nabla \cdot \vec{D} = \rho_f/;/;/;\vec{D} = \epsilon_0\vec{E} + \vec{P} = \kappa\epsilon_0\vec{E}. \tag{50}$$

Also, recall the relation between bound charge and polarization,

$$\nabla \cdot \vec{P} = -\sigma_b. \tag{51}$$

Use the divergence theorem on the above equations to apply the elementary Gauss' Law to the region,

$$\oint \vec{D} \cdot dA = \int \sigma dA \Rightarrow D = \sigma. \tag{52}$$

$$\oint \vec{P} \cdot dA = -\int \sigma_p dA \Rightarrow P = -\sigma_p. \tag{53}$$

But, since $\vec{D} = \epsilon_0\vec{E} + \vec{P} = \kappa\epsilon_0\vec{E}$, one has $D = \sigma = \kappa\epsilon_0 E \Rightarrow E = \frac{\sigma}{\kappa\epsilon_0}$. Plug that into the other relation for D (and use the result for P from above) to get, $\sigma = \frac{\sigma}{\kappa} - \sigma_p$. Thus, $\sigma_p = \sigma\frac{1-\kappa}{\kappa}$, as in choice (E).

Problem 55 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=55*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Fermi Temperature*

When one deals with metals, one thinks of Fermi branding—i.e., stuff like Fermi energy, Fermi velocity, Fermi temperature, etc. So, $\frac{1}{2}v_F^2 \approx kT_F$. Fermi stuff is based on the Fermi-Dirac distribution, which assumes that the particles are fermions. Fermions obey the Pauli-exclusion principle. (c.f. Bose-Einstein distribution, where the particles are bosons, who are less discriminating and inclusive than fermions. Both Bose-Einstein and Fermi-Dirac assume *indistinguishable* particles, but the Boltzmann distribution, which assumes the particles are *distinguishable*)

All the other choices are too general, since bosons can also satisfy them. (Moreover, the Born approximation is pretty much the fundamental assumption of all of QM—every single calculation you do involving the interpretation of mod square of wave functions as probability depend on the Born approx!)

Problem 56 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=56*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Expectation Value*

Recall the following result of the Born Assumption,

$$\langle Q \rangle = \psi^* Q \psi = \langle \psi | Q | \psi \rangle, \tag{54}$$

where the above yields the average value of Q—i.e., the expectation value.

Problem 57 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=57*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Operators*

One doesn't really need QM to solve this. Just plug and chug each of the five functions into the following equation,

$$-i\hbar \frac{\partial}{\partial x} f = \hbar k f \tag{55}$$

Applying the operator $-i\hbar\frac{\partial}{\partial x}$ to each function included in the choice, one gets:
(A) $i\hbar \sin(kx)$ ... which isn't an eigenfunction
(B) $-i\hbar \cos(kx)$ ... which isn't an eigenfunction
(C) $-\hbar k f$ ... eigenvalue is off by a sign
(D) $\hbar k$ ... this is the wanted eigenvalue!
(E) $-i\hbar k$ ... off by a sign and imaginary term. Moreover, operators representing observables in QM have real eigenvalues.

Problem 58 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=58*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Holograms*

A hologram is produced from interference of light. Interference of two-beams (one going directly to the film, the other bouncing off the object) produced by a beam-splitter allows the film to record both intensity and relative phase of the light at each point. Intensity (to wit: $I \propto E_0^2 \langle \sin^2 \omega t \rangle = E_0^2/2$) does not depend on angular frequency, but only on phase and amplitude. Thus, choice (B) is right.

Problem 59 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=59*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Wave Phenomenon → *Group Velocity*

Group velocity is $v_g = d\omega/dk$. So, take the derivative of the quantity $\omega = \sqrt{c^2k^2 + m^2}$ to get

$$v_g = d\omega/dk = \frac{c^2 k}{\sqrt{c^2k^2 + m^2}}. \tag{56}$$

Use the above equation to test the 5 choices:

(A) As $k \to 0 \Rightarrow \omega = 0$, not *infinity*. The first condition doesn't work, no need to test the second (don't have to remember L'Hopital's rule).

(B) Wrong for the same reason as (A).

(C) Wrong because v_g approaches 0, not c, as $k \to 0$.

(D) As $k \to \infty$, $v_g \approx \frac{kc^2}{\sqrt{c^2k^2}} = c$, since $m \ll (ck)^2$. So, v_g doesn't tend towards ∞. This choice is wrong.

(E) This is it. The conditions work (and it's the only choice left).

Problem 60 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=60*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Simple Harmonic Motion*

Recall $F = -\frac{\partial V}{\partial x} = -2bx$. Simple harmonic motion has the simple form, $\ddot{x} \propto x$. Thus $F = m\ddot{x} = -2bx \Rightarrow \ddot{x} = -\omega^2 x$, where $\omega^2 = 2b/m$ is the frequency squared. Thus, simple harmonic motion occurs with a frequency determined by both b and m. This is choice (C).

Problem 61 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=61*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Momentum*

One can easily derive the formula for rocket motion,

$$p_{initial} = p_{final} \tag{57}$$

$$mv = (m - dm')(v + dv) + (dm')(v - V) \tag{58}$$

$$mv = mv + mdv - dm'v - dm'dv + dm'v - dm'V \tag{59}$$

$$mdV \approx dm'V \tag{60}$$

$$mdV = -dmV, \tag{61}$$

where m is the mass of the rocket and m' is the mass of the fuel rejected. v is the initial velocity of the rocket. V is the velocity of the exhaust. The final line comes about from realizing that $dm = -dm'$.

The derivation starts with the assumption that the final mass of the rocket is its original mass minus dm', the eject mass, and that its final velocity is a tiny bit faster than it was before dv. *The exhaust mass' velocity relative to the rocket is V.* (Higher order terms such as $dxdy$ have been thrown out to arrive at the final differential equation.)

V is equivalent to the velocity of the exhaust mass relative to the rocket, the inertial reference system. Thus $u = V$ and the answer is choice (E).

(Note that firing the exhaust backwards generates a forward thrust for the rocket—motion in space is quintessentially dependent on the phenomenon of farting. So, if one is well-equipped and wanna get going in space, just let out a bit of gas, and one's good to go.)

Problem 62 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=62*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Separable Differential Equations*

Solve $m\frac{dv}{dt} = -u\frac{dm}{dt}$. Separate variables to get
$\int u\frac{dm}{m} = \int -dv \Rightarrow u\ln(m_0/m) = v(t)$. None of the answer choices such a ln relation for v, and thus the answer is (E), none of the above.

Problem 63 http://grephysics.net/disp.php?yload=prob&serial=1&prob=63

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Method of Image Charge*

The boundary condition at the conducting plane is $V(0) = 0$. This doesn't mean that one can't put an "image charge" a distance $-D$ away on the other side of the plane to make the calculation easier. Making the directest straight line from the charge to the plane along the z axis, one gets the following image-charge potential:

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{\sqrt{x^2+y^2+(z-d)^2}} - \frac{q}{\sqrt{x^2+y^2+(z+d)^2}}\right). \tag{62}$$

The charge density for a grounded conducting plane is given by $\rho = -\epsilon_0 \frac{\partial V}{\partial n}$, where n is the direction of the directest straight line. The derivative is,

$$\frac{\partial V}{\partial z} = \frac{1}{4\pi\epsilon_0}\left(\frac{q(z-d)}{(x^2+y^2+(z-d)^2)^1.5} - \frac{q(z+d)}{(x^2+y^2+(z+d)^2)^{1.5}}\right), \tag{63}$$

and thus, after combining terms, the density is

$$-\frac{1}{2\pi}\frac{qd}{(x^2+y^2+(z-d)^2)^{1.5}} \tag{64}$$

The problem wants the surface charge density a distance D away from the point charge. So, plug in $\left(x^2+y^2+(z-d)^2\right)^{1.5} = (D^2)^{1.5}$ to get

$$-\frac{1}{2\pi}\frac{qd}{D^3} \tag{65}$$

Problem 64 http://grephysics.net/disp.php?yload=prob&serial=1&prob=64

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Impedance Matching*

Maximal power transfer occurs when the impedance is matched, as in

$$I(X_g) + I(X_l) = IR, \tag{66}$$

which requires that $X_g = -X_l$, as in choice (C).

Problem 65 http://grephysics.net/disp.php?yload=prob&serial=1&prob=65

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Biot-Savart Law*

One either remembers the field of a circular current loop or one derives it from the Bigot-Savart Law,

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l} \times \hat{r}}{r^2}. \tag{67}$$

One finds that $d\vec{l} \times \hat{r} = dl$, as they're perpendicular. $dB = \frac{\mu_0 I dl}{4\pi r^2}$, and the vertical components of the field cancel, thus there are only the horizontal components (parallel to the axis of the ring). r is the directest distance from a differential element on the wire to a point on the axis, and x is the horizontal distance (along the axis) from the center of the ring to that point. Thus, the integral becomes,

$$\int \frac{\mu_0 I dl}{4\pi} \frac{b}{(x^2+b^2)^{3/2}} = \frac{\mu_0 I b}{2} \frac{b}{(x^2+b^2)^{3/2}}, \tag{68}$$

where $\int dl = 2\pi b$.

So anyway, the field, at a point anywhere along the axis of the loop is $\vec{B} = \frac{\mu_0 I b}{2} \frac{b}{(x^2+b^2)^{3/2}}$.

At a point far, far, away, $x >> b$, and thus $\vec{B} \approx \frac{\mu_0 I b}{2} \frac{b}{x^3} = \frac{\mu_0 I b^2}{2x^3}$.

Set $x = c$, where c is the fixed coordinate of that point, to get that the field is proportional to Ib^2, as in choice (B).

Incidentally, defining the magnetic dipole moment to be $\mu = IA = I\pi b^2$, one finds the field of a magnetic dipole to be $\frac{\mu \mu_0}{2\pi x^3}$. The field far from a current loop is the same as the field of a magnetic dipole.

Problem 66 __________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=66*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *First Law*

Recall that $dQ = TdS = PdV + dU$, where work done by the system is positive and heat input into the system is positive. For constant volume, the equation becomes $dQ = TdS = dU$.

$\Rightarrow \frac{1}{T} = \left(\frac{\partial U}{\partial S}\right)_V$

Problem 67 __________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=67*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Partition Function*

The problem gives three non-degenerate energies, so one can just directly plug this into the canonical(?) partition function to get,

$$Z = \sum_i e^{-\frac{\epsilon_i}{kT}} = 1 + e^{-\frac{\epsilon}{kT}} + e^{-3\frac{\epsilon}{kT}}, \tag{69}$$

where k is the Boltzmann constant, T is the (absolute) temperature.

Since,

$$U = NkT^2/Z \frac{\partial Z}{\partial T} = Nk^2 \left(\frac{\epsilon}{kT^2} e^{-\frac{\epsilon}{kT}} + \frac{3\epsilon}{kT^2} e^{-3\frac{\epsilon}{kT}} \right). \tag{70}$$

For $\epsilon >> kT$, one can expand $e^x \approx 1 + x$, and thus,

$$\frac{U}{Nk^2} \approx \frac{\epsilon}{kT^2}(1 - \epsilon/kT) + \frac{3\epsilon}{kT^2}(1 - 3\epsilon/kT) \approx 4\epsilon, \tag{71}$$

where one throws out the higher order ϵ terms. ($Z \approx 1$ in denominator)

The average energy of each particle is $U/3 = \frac{4}{3}\epsilon$, as in choice (C).

Problem 68 __________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=68*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Relativistic Energy*

The relativistic energy relation is,

$$E = \sqrt{p^2c^2 + mc^4}. \tag{72}$$

As the speed of a particle $v \to c$, $E \to pc$, since by the de Broglie relation, $p = h/\lambda = h\nu/c = E/c$. Plug in the requirement into the energy relation to find that $m \to 0$, as in choice (A).

Problem 69

http://grephysics.net/disp.php?yload=prob&serial=1&prob=69

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Length Contraction*

Things moving relative to a particular rest frame appear shorter, i.e., contracted. Knowing that $\gamma > 1$, one can deduce this relation without remembering much about special relativity (other than the hackneyed phrase *length contraction*) $L_{rest} = \gamma L_{moving}$. So, one has $5 = \gamma 3$ for the car, where it is seen to be moving in the reference frame of the garage. This implies that $\gamma = 5/3 \approx 1.6 = \frac{1}{\sqrt{1-(\beta)^2}}$. Recall the following useful table to memorize, for relating $\beta = v/c$ and γ

$$\begin{aligned} \gamma &\Leftrightarrow \beta && (73)\\ 1.005 &\Leftrightarrow 0.1 && (74)\\ 1.033 &\Leftrightarrow 0.25 && (75)\\ 1.155 &\Leftrightarrow 0.5 && (76)\\ 1.51 &\Leftrightarrow 0.75 && (77)\\ 2.29 &\Leftrightarrow 0.9 && (78) \end{aligned}$$

One has $\gamma \approx 1.6 \Rightarrow \beta > 0.75c$ (but less than c). Choice (C) works best.

Problem 70

http://grephysics.net/disp.php?yload=prob&serial=1&prob=70

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Length Contraction*

One deduced the approximate value of γ in Problem 69 (as well as the equation for length contraction). Thus, $4 = \gamma L_{moving} \Rightarrow L_{moving} = 4 \times 3/5 = 12/5 = 2.4$. This is choice (A).

Problem 71

http://grephysics.net/disp.php?yload=prob&serial=1&prob=71

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Relativity!*

The order of the door opening depends on which reference frame one is in. Whether or not one actually put the relativistic cake in one's mouth to have performed the motion of eating the cake... is all relative.

Observe this table summarizing the results of some trivial calculations of rest lengths,

$$\begin{aligned} ....ownframe &\quad other'sframe && (79)\\ car........5m &\quad 3m && (80)\\ garage.....4m &\quad 2.4m && (81) \end{aligned}$$

So, the car fits in the garage in the garage's frame (the car is $3m$ and the garage is $4m$). But, in its own frame, the car is much too big to fit in the garage (car is $5m$ and the garage is merely $2.4m$). Which frame is right? Relativity puts equal footing to either, and thus one shall never know. Be an agnostic, and one just might live a happy carefree life.

Problem 72 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=72*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Wave Phenomenon → *Group Velocity*

Recall the definition of index of refraction n in terms of the speed of light in vacuum c and the velocity of the light in medium v, $n = c/v$. Since the velocity of light in any medium is $v \leq c$, the condition $n \geq 1$ usually holds. However, even if rock salt has $n > 1$, the wave does not exceed the speed of light. The group velocity can travel faster than the speed of light, and apparently it is the group velocity at work in the equation $n = c/v$.

One can also arrive at this conclusion via MOE (Method of Elimination):

(A) Relativity is a pretty general theory that supposedly applies everywhere. (Newtonian mechanics can be achieved using the proper approximation technique.) X-out this choice.

(B) An x-ray is a specific frequency in the electromagnetic spectrum. Nothing forbids an electromagnetic wave from transmitting signals, and thus this choice is out.

(C) Imaginary mass? If confused, save for last comparison.

(D) Historical precedence shouldn't change the correctness of a theory (at least not in the ideal world ETS lives in)... X-this out.

(E) One recalls that there's a difference between group and phase velocities. Could this difference allow one of them to exceed the speed of light? Probably. In either case, this is a much better choice than the other remaining candidate, choice (C). So, choose this.

Problem 73 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=73*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Thin film*

In order for the thin film layer to be non-reflecting, it must cancel the reflected wavelengths—as in a destructive interference. The change in wavelengths is $\Delta\lambda = \lambda - \lambda/2 = \lambda/2$, since the wave changes phase by π at the interface between air and the coating, and changes phase again at the second interface between coating and glass. (Assume that $n_{air} < n_{coating} < n_{glass}$.) Destructive interference is thus given by a half-integer wavelength change $m\lambda/2$, where the smallest change is $\lambda/2$.

The change in wavelength occurs over twice the thickness t of the coating, thus

$$2t = \Delta\lambda = \lambda/2. \tag{82}$$

This implies that $t = \lambda/4$, as in choice (A).

Problem 74 ______________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=74*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Polarizers*

One might remember the result from optics that the maximum fraction incident between three polarizers with the first and third orthogonal to each other is 1/8. Or, if not, one can derive it rather quickly:

Suppose the incident intensity of the light (before going through any polarizers) is I.

Light going through the first polarizer has the intensity $I_1 = I/2$.

Light going through the second polarizer has the intensity $I_2 = I_1 \cos^2\phi = I/2\cos^2\phi$. ($\phi$ is the angle between the polarizer and the light.)

Light going through the third polarizer has the intensity $I_3 = I_2 \cos^2\theta = I/2\cos^2\phi\cos^2\theta$. ($\theta$ is the angle between the polarizer and the light.)

In order for the intensity I_3 to be max, one can take the derivative with respect to either θ or ϕ. Knowing a priori that the first and third polarizers are orthogonal (at 90 degree angles) to each other, one can rewrite either ϕ or θ in terms of the other. So, $\phi = \pi/2 - \theta$, and thus,

$$I_3 = I/2\cos^2(\pi/2-\theta)\cos^2(\theta) = I/2\sin^2\theta\cos^2\theta = I/4\sin(2\theta). \tag{83}$$

The the derivative to find the maximum, $\frac{dI_3}{d\theta} = I/2\cos(2\theta) = 0$. One finds that $2\theta = \pi/2 \Rightarrow \theta = \pi/4$. This implies that Plug in θ to get $I_3 = 1/8$.

Problem 75 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=75*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Kepler's Third Law*

One recalls (or should memorize now) the famous slogan: *The square of the period is proportional to the cube of the distance.* In equation form,

$$T^2 = kd^3 \tag{84}$$

One does not need to know the proportionality factor to solve this problem. In fact, the problem gives $T = 80$ minutes at $d = R_e$. Thus, $80^2 = kR_e^3 \Rightarrow k = 80^2 minutes/R_e^3$.

There are 24×60 minutes in 24 hours. Plug that into the equation (and the relation for k as determined above) to get,

$$(24\times 60)^2 = 80^2/R_e^3 d^3 \Rightarrow d = (24 \times 6/8)^{2/3} R_e \tag{85}$$

No calculators allowed, so $(18^2)^{1/3}$ is about $400^{1/3}$, which is closest to 7, as in choice (B).

Problem 76 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=76*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Energy*

Recall the formula for inertia $I = \int dmr^2$. A hoop has constant $r = R$, thus the integral is trivial and $I_{hoop} = MR^2$. One can break up the kinetic energy into the pure rolling (about center of mass) and pure translation bit,

$$\begin{aligned} Mgh &= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 && (86)\\ &= \frac{1}{2}MR^2\omega^2 + \frac{1}{2}Mv^2 && (87)\\ Mgh &= MR^2\omega^2 && (88)\\ gh &= R^2\omega^2 && (89)\\ &\Rightarrow \omega = \sqrt{\frac{gh}{R^2}}, && (90) \end{aligned}$$

where one recalls that $v = R\omega$.

Plug the value of angular velocity ω into the equation for angular momentum $L = I\omega$, to get $MR^2\sqrt{\frac{gh}{R^2}} = MR\sqrt{gh}$, as in choice (A).

Problem 77 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=77*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Wave Phenomenon → *Wave Equation*

The problem gives the equation of motion
$m\ddot{x} = -kx \Rightarrow \ddot{x} = -\frac{k}{m}x = -\omega^2 x$, where $\omega^2 = \frac{k}{m}$.

The general equation for a wave propagating in time and oscillating in the x direction is $x(t) = A\sin(\omega t + \phi)$. ($A$ is the amplitude, ω is the angular frequency and ϕ is some phase constant.)

This is also the general solution to the differential equation posed above.

Plug in the condition (given by the problem) that
$x(t) = A/2 = A\sin(\omega t + \phi)$ to get $1/2 = \sin(\omega t + \phi)$. Recalling the unit circle, the angle $\omega t + \phi = \pi/6$.

Plug in the argument into the velocity
$\dot{x} = \omega A\cos(\omega t + \phi) = \omega A\cos(\pi/6) = \omega A\sqrt{3}/2$. Recall that $\omega = 2\pi f$, and thus $\dot{x} = \pi f A\sqrt{3}$, as in choice (B).

Problem 78 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=78*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of energy*

There isn't much to say about this problem other than the fact that it's a conceptual conservation of energy problem. Particle 1 moves at velocity v towards particle 2, initially at rest. EM-potential energy increases as they get closer and closer together—but, energy should still be conserved since no energy is radiated. The potential energy increase comes from the initial kinetic energy of particle 1. Take choice (C). (The other choices are all either too weird or just plain bogus.)

Problem 79 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=79*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Field Lines*

The Maxwell Equation that states "No monopoles" requires that the divergence of the magnetic field be 0, or more elegantly, $\nabla \cdot \vec{B} = 0$. The problem asks for fields that violate this condition, so the condition to look for now is $\nabla \cdot \vec{B} \neq 0$.

(A) $\vec{B} = \pm Const$ ... divergence is 0
(B) $\vec{B} = \pm Const$ ... divergence is 0
(C) this does not explicitly require the divergence to be 0.
(D) this is a non-zero divergence, as the field lines diverge outwards from a source.
(E) this is $\nabla \times \vec{B} \neq 0$... doesn't necessarily state anything about $\nabla \cdot \vec{B}$

Problem 80 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=80*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Gauss Law*

Recall the differential form of Gauss' Law $\nabla \cdot \vec{E} = \rho$. For fields that contain no charges, the equation becomes $\nabla \cdot \vec{E} = 0$. Find the choice with 0 divergence.

(A) $A(2y - x) \neq 0$
(B) $A(-x + x) = 0$ ... so this is it!
(C) $A(z) \neq 0$
(D) $A(yz + xz) \neq 0$
(E) $Ayz \neq 0$

Problem 81 ________________ $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=81$
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type
Electromagnetism → *Faraday's Law*

Recall Faraday's Law,

$$V = -\frac{d\Phi}{dt}, \tag{91}$$

where $\Phi = \vec{B} \cdot d\vec{A}$.

The magnetic field at the center of a loop of current-carrying wire is $\vec{B} = \frac{\mu_0 I}{2r}$, where r is the radius of the loop. (If one forgot this, one can remember it from Ampere's Law.) The magnetic field of the outer loop induces an electric field in the inner loop. The outer loop's B field is $\vec{B} = \frac{\mu_0 I}{2b}$.

The area of the *inner* loop stays constant at πa^2. Thus the flux through it is $\Phi = \frac{\mu_0 I}{2b} \pi a^2$.

The larger loop carries an ac current, given by $I = I_0 \cos \omega t$. Thus,

$$|\frac{d\Phi}{dt}| = \frac{\mu_0 I_0 \omega}{2b} \pi a^2 \sin \omega t, \tag{92}$$

as in choice (B).

Problem 82 ________________ $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=82$
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type
Atomic → *Emission Lines*

(A) The Stern-Gerlach effect has to do with splitting of beams of atoms sent through an inhomogenous magnetic field. Does not have to do with emission spectrum.

(B) The Stark effect has to do with energy shifts via placing atom(s) in an Electric field.

(C) Splitting is often due to electron spin magnetic moments.

(D) The condition is much too rigid.

(E) Splitting of energies means more lines than before. This choice is general enough to be true.

(The effect of energy shifts due to placing an atom in a magnetic field is called the Zeeman Effect, which isn't listed.)

Problem 83 ________________ $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=83$
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type
Atomic → *Spectral Line*

The less dense a gas is, the more precise (thin) its emission lines. So, when one has a dense gas, one expects the spectral lines to be broader—as in choice (C).

Problem 84 ________________ $http://grephysics.net/disp.php?yload=prob\&serial=1\&prob=84$
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type
Atomic → *Spectroscopic Notation*

Given the order in which energy levels are filled in atomic configuration $1s2s2p3s3p4s3d$ and the number of electrons in sodium, one can fill it up like $1s^2 2s^2 2p^6 3s^1$. There is a net spin from the missing electron in the $3s$ valance shell, and thus $S = 1/2$. The valance $3s$ shell has $l = 0$ (and $L = S$), since $s = 0; p = 1; d = 2; f = 3$. $J = l + S = 1/2$. Thus, the form should be $^{2(1/2)+1}S_{1/2} =^2 S_{1/2}$

Not even knowing anything about spectroscopic notation, one can deduce the right answer as well as the general form: $^{2S+1}(L)_{J=l+S}$, where L can be either S, P, D, F... depending on whether $l = 0, 1, 2, 3$...

Problem 85 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=85*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Nuclear Physics*

Pair production only occurs above a certain energy on the order of MeV. Thus, all except choices (B) and (C) remain.

The photoelectric effect is dominant for low energies, so its cross-section must be line (1). Choose choice (B).

Problem 86 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=86*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Coulomb's Law*

(Better classified as the Cavendish-Maxwell Experiment to determine the exponent in Coulomb's Law.)

One wonders why the common sense (and much too trivial) answer, choice (D), isn't right. In searching through one's lower level textbook, one would find accounts of Cavendish's Torsion experiment, which seems to also support choice (D). However, reading up on papers, one finds that the precise determination of the exponent is actually done via a whole different method... which although is an experimental technique, actually serves to illuminate the necessity of the inverse square law—and thus, this problem is not classified as a Lab Technique problem.

Charge up a conducting shell. Put a charge inside, distinctly asymmetrically far from the center but not touching the outer shell. One finds that the charge will not move.

This suggests that the field is 0.

Suppose that the field contribution to the test charge can be broken down into two unequal parts, set apart by a plane through the test-charge. $\vec{E} \propto dS_1/r_1^n - dS_2/r_2^n$. Substituting the solid angle $d\Omega = dS\cos\theta/r^2$, one finds that $\vec{E} \propto \frac{d\Omega}{\cos\theta}\left(\frac{1}{r_1^{n-2}} - \frac{1}{r_2^{n-2}}\right)$. Since the field vanishes for any r_1, r_2, one deduces that $n \to 2$.

For other and more modern methods, see page 95ff, especially page 100 of *http://yosunism.com/inverseSquare.pdf*

Problem 87 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=87*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Diatomic Molecules*

Vibrational energy of a diatomic molecule goes to 0 at low temperatures. Thus, the springy dumbbell would be approximately a rigid dumbbell at a low-enough temperature. The other choices are too specific, and thus (E), the most general, must be it.

Problem 88 ________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=88*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Pressure*

Recall the following results for low temperatures,

$$P_{boson} \propto T^{5/2} \quad P_{fermion} \propto T_F \approx big \quad P_{classical} \propto T, \tag{93}$$

where one realizes that the Fermi temperature, T_F tends to be on the order of thousands for most material, and that the low temperature regime temperatures are definitely far less than the Fermi temperature; one thus has $P_F > P_B, P_C$. Moreover, classical effects occur at around $T = 300K$, while the problem specifies the temperature domain for bosons to be far lower than that, thus one deduces that $P_B < P_C$. Therefore, $P_F > P_C > P_B$.

Problem 89 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=89*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Identical Particles*

According to Griffiths, the proof to this rule comes from QFT,

$$\psi = \frac{1}{\sqrt{2}} \left(\psi_\alpha(x_1)\psi_\beta(x_2) \pm \psi_\beta(x_1)\psi_\alpha(x_2) \right), \tag{94}$$

where it's + for bosons and − for fermions.

The given identical particle wave function contains a plus sign, so the particles must be bosons. Bosons have integer spin (while fermions have half-integer spin). Electrons, protons, and neutrons are all fermions. A positron is just a positive electron, so that is presumably, also, a fermion. Thus, the remaining choice would be deuteron—which is a boson.

Problem 90 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=90*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Particle in a Box*

The particle is in an infinite well (or box, if you will) of length 2a. (It's stuck forever bouncing around between the two walls.)

The number of nodes in the wave function determines the energy level. In this case, there is one load, thus this is E_2. The lowest state would be E_1

$E_n = kn^2$ eV for particle in a box. Given that $E_2 = 2 = 4k$, one determines $k = 1/2$ eV. Thus, $E_1 = k = 1/2$ eV. The answer is thus (C).

Problem 91 ——— *http://grephysics.net/disp.php?yload=prob&serial=1&prob=91*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Bragg Reflection*

Bragg diffraction is basically the wavelength change of a wave impinging two adjacent layers of the crystal. More specifically, it relates the wavelength difference of the incident wave on the top layer to that of the reflected wave on the lower layer. The distance between layers ("lattice planes", in Solid-State-Speak) is d, and the angle of incidence is θ. The change in wavelength is thus a simple geometry problem, the result being the celebrated Bragg Reflection Relation,

$$2d \sin\theta = n\lambda, \tag{95}$$

where n corresponds to the order of the reflection.

The problem supplies the following numbers

$$n = 1 \tag{96}$$
$$d = 3E - 10 \tag{97}$$
$$\theta = 30^\circ \tag{98}$$
$$m_e \approx 10E - 31, \tag{99}$$

and if one recalls the de Broglie relation, $p = h/\lambda$, one has, in general, $n\lambda = nh/mv = 2d\sin\theta$. Plugging in some numbers, one finds that $\frac{6E-34}{10E-31v} = 3E - 10$, since $\sin 30^\circ = 1/2$, and where the approximation scheme to battle the no-calculators-allow requirement is shown. Thus, one has $v = 0.25E7$, which is very close to choice (D).

The problem .

Problem 92 $\overline{}$ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=92*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Perturbation Theory*

One can derive the selection rules by applying the *electric dipole approximation* in time-dependent perturbation theory. The results are the following: $\Delta m = 0, \pm 1$, $\Delta l = \pm 1$. Choices (B) and (C) are (exactly this, thus) immediately out. There is no selection rule for spin, and thus choice (D) is it. (The correction is due to user snim1.)

Problem 93 $\overline{}$ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=93*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Power*

Recall the following basic formulas, $P = IV = Fv$, where P is power, I is current, V is voltage, F is force, and v is velocity. $e = \frac{Work_{done}}{Work_{input}}$, where e is the efficiency, which relates work (and thus power).

The problem gives $e = 1$, $F = \mu N = 100\mu$, $v = 10$, $I = 9$, $V = 120$—where all units are SI.

Thus $e = 1 \Rightarrow P_{done} = P_{input} \Rightarrow \mu N v = IV$. Solve for $\mu = \frac{IV}{Nv} = \frac{9*120}{100*10} = 108/100 \approx 1.1$, as in choice (D).

Problem 94 $\overline{}$ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=94*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *LR Circuit*

Once the switch S is closed, although the initial current through the inductor is 0, the change in current through it is maximal. Recalling that $V = -L\dot{I}$, one realizes that the voltage across A must be non-zero at the start. Thus, plots (C), (D), and (E) are eliminated.

Now, one must decide between choices (A) and (B). Once the circuit reaches equilibrium, i.e., the elements reach their asymptotic values. Specifically, the voltage at A goes to 0, since the inductor has no potential difference across it (to wit $\dot{I} = 0$). Once the switch is opened, the current suddenly changes, and $\dot{I} \neq 0$, thus the inductor has a voltage across it, and the voltage at A becomes nonzero. Because of the diode, this would necessarily have to be a negative voltage. Since there is only R_2 to dissipate the voltage, the magnitude of the voltage once the switch is opened should be bigger than that initially, when the switch is closed at t_0. Choose (B).

Problem 95 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=95*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Carnot Cycle*

The Carnot cycle is the cycle of the most efficient engine, which does NOT have $e = 1$ (unless $T_1 = 0$), but rather, $dS = 0$—this means that entropy stays constant. Choice (C) is thus false.

The efficiency of the Carnot cycle is dependent on the temperature of the hot and cold reservoir. The hot reservoir has decreasing entropy because it gets cooler as the cycle proceeds. From writing down the thermodynamic relations for isothermal and adiabatic paths and matching P-V boundary conditions, one can determine that $Q_1/Q_2 = T_1/T_2$. The efficiency is thus $e = \frac{Q_2 - Q_1}{Q_2} = 1 - Q_1/1_2 = 1 - T_1/T_2$.

Problem 96 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=96*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Perturbation*

Technically, one can solve this problem without knowing anything about perturbation theory. Just remember the useful fact that the eigenfunctions of the unperturbed infinite deep well form a complete set (or complete basis). This means that *any function* can be represented by the old unperturbed infinite deep well. Thus, the solution to the perturbed eigenfunction should look like $\psi_0' = \sum_n a_{0n}\psi_n$.

Also, since the perturbed potential is also symmetrical with respect to the origin (as the original unperturbed potential was, too), one knows that all the odd terms should go to 0. This leaves choice (B).

Problem 97 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=97*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Angular Momentum*

The rotational part of the angular momentum is $L = I\vec{\omega_0}$. The translational part of the angular momentum is $L_{v_0} = m\vec{r} \times \vec{v_0}$. (Note that, according to the diagram, this cross-product points in the other direction to the angular velocity.)

Initially, the angular momentum about the point P is $L = \frac{1}{2}MR^2\omega_0 - L_{v_0} = 0$, since $v_0 = \frac{1}{2}R^2\omega_0^2$ and $\vec{L_{v_0}} = \vec{r} \times \vec{p} = -RMv_0 = -\frac{1}{2}M\omega_0 R^2$. QED.

Problem 98 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=98*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Potential*

Recall that $V = \int \frac{dq}{x} = k\int_l^{2l} \lambda \frac{dx}{x} = k\lambda \ln 2$, where $\lambda = Q/l$ is your usual linear charge density.

Problem 99 ______ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=99*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Positronium*

Interestingly, this is almost an exact repeat of Exam 9677 Prob 12. The reasoning's the same:

The positronium atom involves a positron-electron combination instead of the usual proton-electron combo for the H atom. Charge remains the same, and thus one can approximate its eigenvalue by changing the mass of the Rydberg energy (recall that the ground state of the Hydrogen atom is 1 Rydberg).

Recall the reduced mass $\mu = \frac{m_1 m_2}{m_1+m_2}$, where for identical masses, one obtains $\mu = m/2$. The Rydberg in the regular Hydrogen energy eigenvalue formula $E = R\left(1/n_f^2 - 1/n_i^2\right)$ is $R \propto \mu$, where $\mu \to \mu/2$. Substitute in the new value of the reduced mass to get $E \approx R/2$. $R = -13.6$ eV, and thus $E \approx -6.8$ eV.

Problem 100 _____________ *http://grephysics.net/disp.php?yload=prob&serial=1&prob=100*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Diffraction*

The key equation involved is the (Fraunhoffer) diffraction equation, $\sin\theta = \lambda/d$, where the obvious quantities are used.

For small angle, the equation simplifies to $\theta \approx \lambda/d$.

The blur would be due to both the size of the hole and the diffraction. The arclength of the diffraction is approximated by $d_2 = D(2\theta) = 2D\lambda/d$, where the factor of 2 comes from the fact that the arclength angle is symmetrical about the diffraction axis, thus twice the diffraction angle. Define a blur equation $D' = d + d_2 = d + 2D\lambda/d$. Take the first derivative, with respect to the pinhole diameter, set it to 0, to get $dD'/dd = 1 - 2D\lambda/d^2 = 0 \Rightarrow d = \sqrt{2D\lambda} \approx \sqrt{D\lambda}$, as in choice (A).

This solution is due to:

`http://www.exo.net/~pauld/summer_institute`
`/summer_day3eye_and_brain/pinhole_optimum_size.html`