GR0177 Solutions

Detailed Solutions to the GRE Physics Exam

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Problem 1 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=1*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Acceleration Vector*

This is a conceptual vectors problem.

Tangential: One knows that the velocity of the bob at the endpoints is 0, and that it has maximal acceleration (pulling it back towards the center) at the endpoints. In the center, it has maximal velocity and thus minimal acceleration.

Normal: The normal acceleration varies from 0 at the endpoints to its maximal value at the center. The normal acceleration varies so that the sum of tangential and normal accelerations is a constant.

Problem 2 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=2*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Centripetal Force*

Frictional force is given by $F_f = \mu N$, where N is the normal force.

Centripetal force gives the net force to be, $\sum F = mv^2/r = \mu mg$. The m's cancel out, and one gets $r = v^2/(\mu g)$.

The revolutions per minute ω given in the problem can be converted to velocity by $v = r\omega = r(33.3rev/min)(2Pirad/rev)(min/60s) \approx \pi rm/s$. Plugging this into the equation for r above, one has $r = \pi^2 r^2/(\mu g) \Rightarrow r = \mu g/pi^2 \approx 3/\pi^2 <\approx 1/3$, which would be choice (D).

Problem 3 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=3*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Kepler's Third Law*

Recall Kepler's Third Law stated in its most popular form, *The square of the period is proportional to the cube of the orbital radius.* (Technically, the orbital radius is the semimajor axis of the ellipse.)

Recast that commonsense fact above into equations to get $T \propto R^{3/2}$, as in choice (D).

Problem 4 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=4*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Momentum*

From conservation of momentum, one has $2mv = 3mv' \Rightarrow v' = 2/3v$.

The initial kinetic energy is $1/2mv^2 = mv^2$.

The final kinetic energy is $1/2(3m)v'^2 = 3/2m(2/3v)^2$.

The ratio of the final to initial kinetic energy is 2/3.

The kinetic energy lost in the collision is 1 minus that ratio. Thus, the answer is 1/3, as in choice (C).

Problem 5 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=5*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Degree of Freedom*

Each degree of freedom contributes roughly kT to the average total energy. There are three translational degrees of freedom and 3 vibrational degrees of freedom and 3 rotational degrees of freedom. The total energy is 9kT. Divide that by the number of dimensions to get 3kT.

Problem 6 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=6*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Work*

The work done by an adiabatic process is $W = -\frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$. (One can quickly derive this from noting that $PV^\gamma = const$ in an adiabatic process and $W = \int PdV$.)

The work done by an isothermal process is $W = nRT_i ln(V_2/V_1) = P_iV_i ln(V_2/V_1)$ (One can quickly derive this from noting that $P_2V_2 = P_2V_2 = nRT_1 = nRT_2$ for an isothermal process.)

From the above formulae, one can immediately eliminate choice (A).

One can calculate the isothermal work to be $W_i = nRT_1 ln2 = P_1V_1 ln2$.

One can calculate the adiabatic work to be:

$W_a = \frac{1}{1-\gamma}(P_1 - 2P_2)V_1 = \frac{1}{1-\gamma}(P_1 - 2/2^\gamma P_1)V_1 = \frac{1}{1-\gamma}(1 - 2^{1-\gamma})P_1V_1$.

For a monatomic gas, $\gamma = 5/3$, and one finds that $0 < W_a < W_i$. Choice (E).

(Also, in general, an adiabatic process always does less work than an isothermal process in a closed cycle.)

Problem 7 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=7*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Field Lines*

Since one has same polarity magnet ends at the same side, one can immediately cancel out choices (A) and (D). They would repel each other, and thus the field-lines would not be pointing towards each other. Since there are no such things as magnetic monopoles, one cannot have choice (C), as that would imply a non-zero divergence in the magnetic field. Choice (D) would work if there were a current through the magnets, but there's not. Choice (B) remains. It makes sense since the magnetic charges should repel each other, and thus the field lines should point away showing the repulsion.

Problem 8 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=8*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Conductors*

A conductor has the magical mystical property of inducing the exact opposite charge of a nearby charge. One has all the electrons in the conductor moving towards the positive charge placed outside Q. A net negative charge of -Q is induced.

(Also, one recalls that this was the assumption made in finding the potential of this setup using the Method of Images—the charged plane was assumed to have a charge density $-Q/A$.)

Problem 9 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=9*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Electric Field*

From symmetry, one has 0-field at the center. This is choice (A).

Take the limit of an infinite number of charges placed around the center. In two dimensions, one has a circle of charges. This is similar to a 2-dimensional conductor. There is 0-electric-field inside a conductor (the potential is constant inside). But, for a less-than-infinite number of charges, one would only have 0-field in the center. Gauss' Law inside would have a gaussian surface (just a loop in 2-dimensions) surrounding 0 charge.

The brute-force way would be to compute the electric field via Coulomb's Law and vector addition. With the proper coordinates and a $2\pi/5$ angle unit circle, this is a easy calculation.

Problem 10 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=10*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Capacitors*

The problem gives $C_1 = 3\mu F$ and $C_2 = 6\mu F$ and V=300 V.

The capacitors are connected in series, and thus $C_{eq} = C_1C_2/(C_1 + C_2) = 18/9 = 2\mu F$. (Recall that series and parallel capacitor formulae are opposite to that of resistors.)

Energy is $1/2C_{eq}V^2 = 9E4 \times 1E - 6 = 9E - 2$, as in choice (A).

Problem 11 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=11*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Lensmaker Formula*

The lensmaker's formula is $1/d_o + 1/d_i = 1/f$. For an image on the opposite side of the light, the image distance is taken as positive.

The distance between the object and the first lens is $d_{1o} = 40$. $f_1 = 20$. The lensmaker's formula gives $1/d_{1i} = 1/f_1 - 1/d_{1o} = 1/40$. Thus, the image is 40 cm behind the first lens.

The first image forms the object for the second lens. The distance of the first image to the second lens is $40 - 30 = 10cm$. Since this image is behind the lens (on the other side of the incident geometric light), the convention in the lensmaker's formula takes this as a *negative* distance. One has, $1/d_{i2} = 1/f_2 - 1/d_{o2} = 1/10 + 1/10 = 2/10 = 1/5$. Thus, the second image is 5 cm to the right of the second lens.

Problem 12 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=12*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Mirror Formula*

The mirror formula is identical in form to the lensmaker's formula in Problem 11. The sign convention varies for the image distance. If an image distance is on the inside of the mirror, then it is taken as negative.

Thus, $1/f = 1/d_o + 1/d_i \Rightarrow 1/d_i = 1/f - 1/d_o$. From the diagram, one deduces that $d_o < f$. Thus, one finds that d_i has to be *negative*. The virtual image has to be inside the mirror. Only choice (V) shows this.

Problem 13 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=13*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Aperture Formula*

The circular aperture formula (a.k.a. Rayleigh Criterion) is given by $\theta = 1.22\lambda/D$. Plug in the given quantities to get that.

(It's a nice formula to memorize, as it's used as common sense in a variety of engineering fields, as well as in remote-sensing, such as satellite-communications.)

Problem 14 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=14$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Cross Section Formula*

When the detector is placed near the source, the particles enter it directly from the circle at the end of the cylinder. The area is $A = \pi d^2/4$

When the detector is placed 1 m away, the particles enter it through a sphere of radius 1 m, thus $A = 4\pi R^2 = 4\pi$.

The ratio of areas gives the fraction of detected gamma rays, $d^2/16 = 8^2E - 4/16 = 4E - 4$, as in choice (C).

This is due to Jon Jockers.

Problem 15 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=15$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Lab Methods → *Precision*

The most precise measurement might be the wrong value, as long as after meany measurements, each measurement is very close to the mean. Thus, the width of the height vs number of trials graph must be as thin as possible. The only choice that shows this characteristic is choice (A).

Problem 16 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=16$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Lab Methods → *Sample*

The mean of the ten number is $\bar{x} = 2$. Thus, the standard deviation of the sample is $\sqrt{\bar{x}} \approx 1.44$.
If the student wants to obtain an uncertainty of 1 percent, then
$\sigma/\bar{x'} = 1/100 = \sqrt{2C}/(2C) = 1/(\sqrt{2C})$,
where one assumes the average scales uniformly and C is the time to count.
Thus, one has $\sqrt{2C} \approx 100$. Thus the student should count C=5000 s.

A Sneak Peek of *The Not-So-Boring Review of Undergrad Physics*

Perhaps an alternate description of the Poisson Distribution might help you remember it better. (Note: this section also contains a quick review of the operators of the algebraic approach to the quantum mechanical simple harmonic oscillator.)

The Poisson Distribution is intimately related to the raising and lowering operators of the (quantum mechanical) simple harmonic oscillator (SHO). When you hear the phrase "simple harmonic oscillator," you should immediately recall the number operator $N = a^\dagger a$, as well as the characteristic relations for the raising $a^\dagger$ and lowering a operators, $a|n\rangle = \sqrt{n}|n-1\rangle$ and $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$. And, don't forget the commutation relations that you should know by heart by now, $[a, a^\dagger] = 1$. (That's all part of the collective consciousness of being a physics major.)

Now, here's some quasi-quantum magic applied to the Poisson Distribution. I'm going to show you how to arrive at the result for standard deviation, i.e., $\Delta n \equiv \sqrt{\bar{n}}$ from using the SHO operators.

Let's start with something easy to help jog your memory: The mean or average number in the distribution is just the expectation value of the Number operator, $\langle N\rangle = \langle n|N|n\rangle = \langle n|a^\dagger a|n\rangle \equiv \bar{n}$

Okay! So, on with the fun stuff: the standard deviation is given by the usual definition, $(\Delta n)^2 = \langle N^2\rangle - \langle N\rangle^2$.

The second term is already determined from the above expression for the mean, $\langle n\rangle^2 = \bar{n}$.

The first term can be calculated from $\langle N^2\rangle = \langle n|a^\dagger a a^\dagger a|n\rangle$. Now, the commutation relation gives, $[a, a^\dagger] = 1 = aa^\dagger - a^\dagger a \Rightarrow aa^\dagger = a^\dagger a + 1$. Replacing the middle two of the four a's with that result, the expression becomes $\langle N^2\rangle = \langle n|a^\dagger(a^\dagger a + 1)a|n\rangle = \langle n|a^\dagger a^\dagger aa + a^\dagger a|n\rangle = \langle|a^\dagger a^\dagger aa|\rangle + \langle|a^\dagger a|\rangle = \langle N^2\rangle + \langle N\rangle = \bar{n}^2 + \bar{n}$.

Plugging the above results into the standard deviation, I present to you, *this*: $(\Delta n)^2 = \bar{n}^2 + \bar{n} - \bar{n} = \bar{n}^2 \Rightarrow (\Delta n) = \sqrt{\bar{n}}$.

It's no coincidence that the above works.

The Poisson Distribution is given by $P(\lambda) \propto e^{-m}\frac{m^\lambda}{\lambda!}$. But, wait, doesn't that look a wee bit too familiar? The energy eigenfunctions of the SHO is given by $|n\rangle = \frac{(a^\dagger)^n}{(n!)^{1/2}}|0\rangle$, where $|0\rangle \propto e^{-a}$. Making the following associations, $m \to a$ and $\lambda \to n$, you carve the first etchings in the Rosetta Stone between probability and quantum mechanics.

Hmmm... so are you beginning to see the greater meaning of the magic math of quantum mechanics that you might have learned by rote? Yes, my padawan, I see the force in you.

Problem 17 *http://grephysics.net/disp.php?yload=prob&serial=3&prob=17*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Orbital Config*

First, Check that all choices have superscripts that add to 15. That's apparently so, so one can't easily eliminate choices based on that.

Since the problem has electrons in ground-state, choice (D) and (A) cannot be it–no electrons are promoted from the s to higher orbitals. s has to be filled first.

Choice (E) skips the 3s states completely, so that's out.

One should have an approximate idea of what the periodic table looks like. The transitional (d-state) elements do not occur until *after* 4s. Even so, the first row of transitional elements is labeled 3d. Since Phosphorous does not have enough electrons to qualify for a 4s or more state, choice (B) must be right.

Problem 18 *http://grephysics.net/disp.php?yload=prob&serial=3&prob=18*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Bohr Theory*

One can calculate the ionization energy from finding the ionization energy for He for He+, then subtracting it from the given ionization energy. Since He+ is a hydrogenic atom, one can apply Bohr Theory.

In Bohr Theory, the energy to remove an electron is $E = Z^2E_1$. For Helium, since it has two protons, $Z = 2$. Thus, $E = 4E_1 \approx 52$ is the ionization energy. ($E_1 = 13.6eV$ is just 1 Rydberg or the energy of the ground state of Hydrogen.)

Subtract this from the ionization energy for He given in the problem to be 79 eV, to get an answer close to choice (A).

Problem 19 *http://grephysics.net/disp.php?yload=prob&serial=3&prob=19*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Astrophysics*

It takes 4 H's to create a Helium nucleus in the sun's primary thermonuclear reactions. One either remembers this or can derive it from conservation of mass. The atomic mass of Hydrogen is 1 (since it has just 1 proton), while the atomic mass of Helium is 2 (2 protons, 2 neutrons).

Problem 20 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=20*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Bremsstrahlung*

One might recall the English translation of Bremsstrahlung, which is "braking radiation." The only choice that has to do with acceleration is choice (E).

Problem 21 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=21*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Bohr Formula*

One applies the Bohr formula $E \propto (1/n_f^2 - 1/n_i^2) \propto 1/\lambda$. For the Lyman radiation, this is $E_L \propto (1 - 1/4)$. For the Balmer radiation, this is $E_B \propto (1/4 - 1/9)$. Take the ratio of that to get $E_L/E_B = \frac{3/4}{5/36} = 27/5$. But, since the problem wants the ratio of the *wavelengths*, which has an inverse relation to energy, one takes the inverse of that to get choice (B).

Problem 22 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=22*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Orbits*

The planet orbits in an ellipse, and thus the sum of the minimum and maximum distance is twice the semimajor axis of the orbit. Choice (E) is out.

From Kepler's third law (which can be derived from the formalism of central force theory), one has $T^2 \propto r^3$. Thus, knowing the distances, one can find the period. Choice (D) is out.

The speed can be found from conservation of energy. For an elliptical orbit, one has $E = -k/(2a) = 1/2mv^2 - k/r$, where $k = GmM$ and a is the semimajor axis. (One can find the total energy of the orbit from the Virial Theorem. Recall that $\langle T \rangle = -\langle \sum F \cdot r \rangle/2$. Plug in the gravitational force $F = k/r^2$ to get $\langle T \rangle = \langle k/2r \rangle = \langle k/2a \rangle$. Thus, $T + V = k/2a - k/a = -k/2a$.)

The full form of Kepler's third law is $\tau^2 \approx a^3/(GM)$. Thus, one can determine the mass of the planet.

The only one that remains is choice (A).

Problem 23 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=23*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Vectors*

If a particle is moving in a circle, then its acceleration points towards the center—even if it is moving at a constant (magnitude) velocity. This is the normal component of acceleration.

Now, if its tangential velocity is increasing, then it has a tangential acceleration. The tangential acceleration is given as just $a_t = 10m/s^2$.

The normal acceleration is given by the centripetal acceleration formula $a_n = v^2/r = 100^2/10 = 10$.

Vectorially add the normal and tangential acceleration and dot it with the tangential velocity to find that the angle between them is just 45 degrees.

Problem 24 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=24*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Kinematics*

If a stone is thrown at such an angle at an initial velocity, its horizontal v_x vs t graph should be constant and positive $v_x = v_{x0} = v_0 \cos(45°)$. Thus, choices (A) and (E) are out.

Recalling the basic kinematics equation $v_y = v_{y0} - gt$, one eliminates choice (D), since that shows a parabolic time dependence, when a linear one is required. Since the slope is negative, the v_y-graph should look like III one has choice (C).

(If one forgets the basic equations above, one can derive it all from summing up the net force $\ddot{y} = -g$. Integrate both sides to get velocity. Integrate again to get position.)

Problem 25 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=25*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Moment of Inertia*

The moment of inertia of the center penny about the center is just $1/2mr^2$

The moment of inertia of any one of the other pennis about the center is given by the parallel axis theorem, $I = I_{CM} + md^2$, where d is the distance from the new point from the center of mass. $I_{CM} = mr^2/2$ for each penny, and thus one has $I = mr^2/2 + md^2 = mr^2/2 + m(2r)^2 = 9/2mr^2$, since the distance from the center of each penny to the center of the configuration is 2r.

Since there are 6 pennies on the outside, one has the total inertia $I = 1/2mr^2 + 54/2mr^2 = 55/2mr^2$, as in choice (E).

Problem 26 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=26*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Energy*

Conservation of energy gives, $E = MgL/2 = 1/2Mv^2 + 1/2I\omega^2$, where the potential energy is given about the center of mass. The moment of inertia is about the center of mass, too. The translational velocity is the velocity at the center of mass.

For a rod, one has $I = 1/12ML^2$. $v = \omega L/2$ Plug stuff in to get $MgL/2 = 1/2M\omega^2/4 + 1/2(1/12ML^2)\omega^2 = 1/2(1/4 + 1/12)ML^2\omega^2 \Rightarrow g = (1/4 + 1/12)L\omega^2 = (1/3)\omega^2 \Rightarrow \omega = \sqrt{3g/L}$. Now that one has the angular velocity, one can calculate the velocity at the endpoint; the rod rotates about its other end on the ground. $v = \omega \times r = L\omega = \sqrt{3gL}$, which is choice (C).

Problem 27 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=27*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Hermitian Operator*

The eigenvalues of a Hermitian operator are *always* real. This is one of those quantumisms one memorizes after even a semi-decent course in QM.

One can quickly prove it by the other quantumism-proof one memorized for that same course: Suppose one has $A|\psi\rangle = a|\psi\rangle$. Then, $A^\dagger|\psi\rangle = a^*|\psi\rangle$.

$A|\psi\rangle - A^\dagger|\psi\rangle = (a - a^*)|\psi\rangle$. But, the left side is just 0 from, as $A^\dagger = A$ from the definition of Hermitian operator. Then, the right side must be 0, too, since $\langle\psi|\psi\rangle \neq 0$ in general.

Problem 28 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=28*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Orthogonality*

States or orthogonal when their dot-product (bra-ket) is 0. Since the eigenvectors $|i\rangle$ are given as orthonormal, $\langle i|i\rangle = 1$.

$\langle\psi_i|\psi_2\rangle = 5 - 15 + 2x = 0$ Solve the equation to find that $x = -10$, as in choice (E).

Problem 29 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=29*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Expectation Value*

$\langle O\rangle = \langle\psi|O\psi\rangle = -1/6 + 1/2 + 2/3 = 1$.

(Recall that $\langle\psi|A|\psi\rangle = a\langle\psi|\psi\rangle = a$.)

Problem 30 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=30*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Hydrogen Atom*

The hydrogen atom wave-functions all have to do with an exponentially decreasing radial function. Thus, choose choice A. (FYI: Only the spherical harmonic angular parts have trig involved, and thus choice.)

Problem 31 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=31*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Positronium*

The positronium atom ground-state energy can be deduced from recalling the reduced mass. $\mu = m/2$, since one has a positron and electron orbiting each other, and thus the energy, which is related to mass, is half of that of Hydrogen. $E_{positronium} = -13.6/2eV = -6.8eV$

The Bohr formula still applies, and thus $E = E_1\left(1/n_f^2 - 1/n_i^2\right) = 8E_1/9$, which is choice (A).

Problem 32 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=32*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Momentum*

The total energy of a particle of mass m is $E = 2mc^2 = \gamma mc^2$, since the problem supplies that it is equal to twice the rest mass. This means that $\gamma = 2$.

The relativistic momentum is $p = \gamma mv = 2mv$. However, $v \neq c$.

Knowing γ, one can quickly solve for $\gamma = 2 = 1/\sqrt{1-\beta^2}$. One finds that $\beta^2 = 3/4$. Thus, the velocity is $v = \sqrt{3}/2c$. Plug this into the momentum equation to get, choice (D).

Problem 33 ______ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=33*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Frames*

The distance the pion must travel is in the lab frame, and thus $L = 30 = vt$.

The decay time is given in its proper time, and thus $t = \gamma t_0 \Rightarrow 30 = v\gamma t_0$. Solving for v, one finds that choice (D) works.

Problem 34 ______ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=34*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Spacetime Interval*

The spacetime interval for the convention $ds^2 = c^2dt^2 - dx^2$ has regions where the slope is greater than that of the lightcone line, i.e., $ct = x$, to be timelike. Slopes that are less than the lightcone slope on the plot of x (horizontal) vs ct (vertical) correspond to spacelike phenomena. In this region, one can have two observers disagree on whether an event happens before the other. Thus, one wants $ct < x$, as in choice (C).

Problem 35 ______ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=35*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Blackbody Radiation Formula*

The blackbody radiation formula has $u = \sigma T^4$. The ratio of energy radiated is thus T_2^4/T_1^4. Since $T_2 = 3T_1$, the ratio is just $3^4 = 81$, as in choice (E). (Obviously an increase since the final temperature is much higher than the initial.)

Problem 36 ______ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=36*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Adiabatic*

In an adiabatic expansion, $dS = 0$ (entropy), since $dQ = 0$ (heat). No heat flows out, by definition, and thus choice (A) is out, as well as choice (B). Choice (C) is true since by the first law, one has $Q = U + W \Rightarrow U = -W$, and the given integral is just the definition of work. Choice (D) defines work. Choice (E) remains—so take that.

Problem 37 ______ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=37*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Cycle Analysis*

For path C to A, one has $W = 0$ for an isochoric (constant volume) process.

For path A to B, one has just $W = P\Delta V = 200(V_B - 2)$ for an isobaric (constant pressure) process.

For path B to C, one has $W = P_c V_c ln(V_C/V_B) = 1000 ln(2/V_b)$.

One can figure out V_B from the isothermal condition $P_C V_C = P_B V_B \rightarrow V_B = P_C/P_B V_C = 5/2 \times 2 = 5$. Plug that in above to get,

$W_{CA} = 0$

$W_{AB} = 600$

$W_{BC} = 1000 ln(2/5) > -1000$

$\sum W \approx -400$, which is closest to choice (D).

Problem 38 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=38*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *AC Circuit*

The current is maximized when one has resonance. Resonance occurs when the complex impedance is 0, or when $X_L = X_C \Rightarrow \omega L = 1/(\omega C)$. Plug stuff in to get choice (D).

Problem 39 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=39*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Filters*

High-pass filters have, for high-frequencies ($\omega \rightarrow \infty$), $V_{in} \approx V_{out}$.

The net impedance is given by $Z = R + j(\omega L - 1/(\omega C))$. $V_{in} = IZ$, and thus $I = V_{in}/Z$.

For cases I and II, $Z = R + j(\omega L)$.

For case I: $V_{out} = IR = V_{in}R/(R + j(\omega L))$ Thus, in the regime of high, frequency, one gets $V_{out} \rightarrow 0$ (This is a low-pass filter.)

For case II: $V_{out} = IZ_L = V_{in}\omega L/(R + j(\omega L))$, one gets $V_{out} \approx V_{in}$ (To wit: L'Hopital's Rule can be used or this limit.)

For cases IV and III, $R + j(-1/(\omega C))$.

For Case III, $V_{out} = IR = V_{in}R/(R + j(-1/(\omega C)))$ For high-frequency, $1/(\omega C) \rightarrow 0$, and thus one has $V_{in} \approx V_{out}$.

For Case IV, $V_{out} = IZ_C = V_{in}X_C/(R + j(-1/(\omega C)))$. This quantity goes to 0 for high-frequency. (This is a low-pass filter.)

Hence, the only choices that work for high-freq filters are choices II and III. Choice (D).

Problem 40 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=40*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *LR Circuits*

Immediately after the switch is closed, the voltage across the inductor is maximal since the change in current is huge. (To wit: $V_L = \dot{I}L$) Thus, only choices (D) and (E) remain.

The voltage exponentially decreases rather quickly in a LR circuit, and thus one should choose choice (D) instead of (E).

(One can write out the equation $\ddot{Q}L + \dot{Q}R = V$ and solve for V to verify.)

Problem 41 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=41*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Monopoles*

Theorists like monopoles because if they exist, Maxwell's equation would be symmetrical. Thus, one would get to add terms to the divergence of B and the curl of E to make it symmetric with their other-field counterparts. Choose choice (E).

Problem 42 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=42*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Faraday Law*

Faraday's law suggests that the current inducted opposes the changing field. (This is also known as Lenz Law). It acts something like an electromagnetic inertia.

Since the middle loop is moving towards the observer, the loop A feels an increasing current. Loop A will thus compensate with a current that acts to decrease the field-change; this current is counter-clockwise by the right-hand rule.

Since the middle loop is moving away from loop B, loop B will want to increase its magnetic field, and thus its current is in the same direction as the middle loop.

Only choice (C) works.

(One can immediately cancel out all but choices (A) and (B) from just a conceptual understanding of Lenz Law.)

Problem 43 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=43*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Commutators*

Write it out and recall the commutator identities $[B, AC] = A[B,C] + [B,A]C$ and $[A,B] = -[B,A]$.

Thus $-[L_z, L_x L_y] = -L_x[L_z, L_y] - [L_Z, L_x]L_y = i\hbar(L_x^2 - L_y^2)$, using the identities supplied by ETS.

Problem 44 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=44*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Measurements*

When one measures something, one can get something that is the eigenvalue of the operator. Energy eigenvalues are proportional to n^2. The only choice that is a squared quantity is choice (D), for $n = 3$.

Problem 45 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=45*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Simple Harmonic Oscillator*

The expectation value is the average value, and it is calculated by $\langle\psi|H\psi\rangle$. Using orthonormality, one has $E = (1/14)(3/2) + (4/14)(5/2) + (9/14)(7/2)\hbar\omega = 43/14\hbar\omega$

Problem 46 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=46*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *de Broglie Wavelength*

Since $E = p^2/(2m)$ and the de Broglie wavelength gives $\lambda = h/p$, one can find the initial wavelength $\lambda = h/sqrt2mE$. This yields an expression for $h = \lambda\sqrt{2mE}$

When the particle enters the well, its energy becomes $E^{'} = E - V$, and thus $\lambda^{'} = h/sqrt2m(E - V)$.

Plugging h in, one has $\lambda^{'} = \lambda\sqrt{2mE}/sqrt2m(E - V)$, which is choice (E).

(This above solution is due to the lout Marko.)

Also, the current author's original approach leads to the right result, but it hand-waves the massiveness of the particle:

The initial kinetic energy of the free particle is its total energy $E = hv/\lambda \Rightarrow \lambda = hv/E$. But, since one's dealing with de Broglie, one has $p = h/\lambda = mv$. Thus, $\lambda^2 = h^2/(mE)$.

In the potential, its energy becomes $E - V$. Replace E above with that to get $\lambda^{'2} = h^2/(m(E - V))$. But, from the same relation above, one has $h^2 = mE\lambda^2$. Thus, $\lambda^{'2} = \lambda^2 E/(E - V) = \lambda^2(1 - V/E)^-1$. This is choice (E).

Problem 47 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=47*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Entropy*

Entropy is given as $dS = \int dQ/T$. Since the volume expands by a factor of 2, the work in the isothermal process is $W = nRTln(V_2/V_1) = nRTln2$. But, for an ideal gas, the internal energy change in an isothermal process is 0, thus from the first law of Thermodynamics, one has $dQ = dW + dU \Rightarrow dQ = dW$. The temperature cancels out in the entropy integral, and thus the entropy is just $nRln2$, as in choice (B).

Problem 48 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=48*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *RMS Speed*

In case one forgets the RMS speed, one does not need to go through the formalism of deriving it with the Maxwell-Boltzmann distribution. Instead, one can approximate its dependence on mass and temperature by $3/2kT = 1/2mv^2$. One thus has $v \propto \sqrt{kT/m}$. For the ratio of velocities, one has $\frac{v_N}{v_O} = \frac{m_O}{m_N}$. Plug in the given molecular masses for Oxygen and Nitrogen to get choice (C).

Incidentally, the trick for memorizing the diatomic gasses is *Have No Fear Of Ice Cold Beer* (Hydrogen, Nitrogen, Florine, Oxygen, Iodine, Chlorine, Bromine).

Problem 49 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=49*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Partition Functions*

The partition function is given by the formula $Z = \sum_i g_i e^{-\epsilon_i/kT}$, where g_i enotes the degeneracy of the ith state.

Plug in the given information into the formula to get choice (E).

Problem 50 — *http://grephysics.net/disp.php?yload=prob&serial=3&prob=50*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Wave Phenomena → *Sound*

Since the wavelength of the wave does not change, as the pipe presumably stays approximately the same length, only the frequency varies. If the speed of sound changes, then the frequency changes. If the speed of sound is lower than usual, then the frequency is lower. Thus, choices (A), (B) and (C) remain. Calculate 0.97×440 to get choice (B).

Problem 51 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=51*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Polarizers*

When one has three polarizers with the first oriented at a 90 degree angle to the last, the maximum light transmitted is $I_0/8$.

In this case, the intensity of light transmitted through the first filter is $I_0/2$, where I_0 is the incident light. (Half the light has been canceled by the polarization.)

The intensity of the light transmitted through the second filter is $I_2 = |I_1 \cos 45°|^2 = I_0/4$.

The intensity of the light transmitted through the third filter is $I_3 = |I_2 \cos 45°|^2 = I_0/8$.

This is choice (B).

Problem 52 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=52*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Solid State Physics*

Recall the definition of a primitive cell to be the unit cell divided by the number of lattice points in a Bravais lattice. A Bravais lattice is just a lattice that looks isotropic from any point—everywhere the same no matter the point-perspective.

Simple cubic has 1 lattice point to generate its Bravais lattice.

Body-centered cubic has 2 lattice points to generate its Bravais lattice. (One can keep on tesselating the a lattice point on one corner and the lattice point on the body center to generate the whole BCC lattice.)

Face-centered cubic has 4 lattice points to generate its Bravais lattice. (One can keep on tesselating the lattice point on each face surrounding a corner lattice point.)

So, anyway, from the above, one finds choice (C) for BCC's unit cell.

Problem 53 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=53*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Solid State Physics*

From *Ibach and Luch*, one finds that the resistivity of a semiconductor varies as $1/T$.

(Also, from elementary electrodynamics, one recalls the resistivity equation $\rho(T_2) = \rho(T_1)(1 + \alpha\Delta T)$. Semiconductors have a *negative* coefficient of resistivity α, and thus the resistivity should decrease with increasing T. The only graph that shows this behavior of decreasing resistivity with T is (B).)

Problem 54 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=54*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Impulse*

Impulse is defined as $J = \int F dt$. But, for this problem, one doesn't have to evaluate a messy integral. Instead, the area under the curve is just the sum of two triangles. $J = 2\frac{1}{2}(2)(1) = 2kgm/s$, as in choice (C).

Problem 55 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=55*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Momentum*

From conservation of momentum in the horizontal direction, one has $mv = 2mv' \cos\theta$. Solving for the final velocity, one has $v' = \frac{v}{2\cos\theta}$. Since $|\cos\theta| < 1$ for $\theta > 0$, one finds that $v' > v/2$, as in choice (E).

Problem 56 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=56*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Fluid Statics*

The physical equation required is the buoyancy equation or Archimedes' Principal. The buoyant force is given by, $F_{buoyant} = \rho_{fluid} g V$. ρ_{fluid} refers to the density of the fluid that buoy's the object. In this case, $\rho_{fluid} = \rho_a$ is just the density of air (not Helium).

The buoyant force for this problem is $F_b = \rho_a g V_{He}$. This force must balance the load carried by the balloon. One now has (approximately, to simplify the calculations, since one is neglecting the weight of Helium) a simple sum of the forces problem, $\sum F = F_b - mg = 0 \Rightarrow 300g = \rho_a g V_{He}$.

Now, solving for the volume of Helium required, one has $V_{He} = 300/\rho_a$. Since $\rho_a = 1.29$, this is about $230m^3$. However, since the balloon has to lift the weight of the Helium as well, the actual volume should be slightly larger. The closest choice is (D).

Problem 57 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=57*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Dimensional Analysis*

The only choice that has the right unit for force is choice (A), since the expression has the units $(kg/m^3)(m/s)^2(m^2) = N$.

Problem 58 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=58*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Trajectory*

The velocity of the proton as it enters the region is given by its acceleration due to the potential difference V. Conservation of energy yields $1/2mv^2 = qV \Rightarrow v^2 = 2qV/m$.

In the first experiment, since the electric and magnetic forces balance, as the particle is un-deflected, one deduces that $qvB = qE \Rightarrow vB = E$ from the Lorentz force.

In the second experiment, the potential is doubled to 2V. The velocity is thus greater than it was in the first experiment. Since the magnetic force is proportional to the velocity, one has the magnetic force larger. Thus, the particle is deflected in the direction of the magnetic field, which is in the -x-direction from the right hand rule, as in choice (B).

Problem 59 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=59$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *LC Circuit*

The form for a simple harmonic oscillator is $m\ddot{x} + kx = 0$, which one can obtain from Hooke's Law. Comparing this to the LC circuit equation $L\ddot{Q} + Q/C = 0$, one sees that $L \Leftrightarrow m$ and $1/C \Leftrightarrow k$ and $Q \Leftrightarrow x$. This is choice (B).

Problem 60 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=60$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Flux*

Electric flux is given by $\oint \vec{E} \cdot d\vec{A}$. The electric field from the infinite conducting plane is σ/ϵ_0. The field points directly perpendicular to the plane of the conductor, and thus the normal area it fluxes-through in the Gaussian surface is just $\pi(R^2 - x^2)$. (This geometry is obvious if one draws a right-triangle with the bottom leg of length x, the hypotenuse of length R, and the side-leg bordering the conductor.)

Plug everything into the flux equation to get choice (D).

Problem 61 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=61$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Boundary Conditions*

For a conductor, the electric field boundary condition at the interface are $E_1^\perp - E_2^\perp = \sigma_f$ (σ_f is the free charge density) and $E_1^{||} - E_2^{||} = 0$.

The plane wave traveling in the x-direction is polarized (say) in the +z-direction. Thus, $\vec{E} = \hat{z}E_0\cos(kx - \omega t)$.

There is no component perpendicular to the conductor at the boundary, and thus the first boundary condition implies that the free charge density is 0.

The second boundary condition requires that $E_1^{||} = E_2^{||}$. The parallel component of E is polarized in the z-direction, and thus the requirement is $E_0 + E_0^r = E_0^t$, i.e., the incident plus the reflected is equal to the transmitted wave. However, for a perfect conductor, the transmitted wave is 0. Thus, one has $E_0 = -E_0^r$. This implies that the electric field to the left of the conductor *cancels*.

Recall the Poynting vector, $\vec{S} \propto \vec{E} \times \vec{B}$, which conveniently *points* in the direction of the electromagnetic wave propagation. Since the electric field (by the convention used above) is polarized in the z-direction, the magnetic field of the incident wave points in the -y direction. However, since the electric field of the reflecting wave points in the -z-direction, its magnetic field also points in the -y direction. The magnetic field magnitude is thus $2E_0/c$. Hence, one has choice (C).

Problem 62 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=62$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Cyclotron Frequency*

The cyclotron frequency can be easily derived by equating centripetal force with the Lorentz force. It applies to this problem since the magnetic field is perpendicular to the velocity of the particle. $qvB = mv^2/r = mv\omega \Rightarrow \omega = qB/m$, where one recalls the angular frequency $\omega = v/r$.

Solving for m in the cyclotron frequency expression above, one has $m = qB/(2\pi f)$, where one recalls that the angular frequency is related to the frequency by $\omega = 2\pi f$.

Plug in the numbers to get choice (A).

Problem 63 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=63*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Wien Law*

Recall Wien's Law, $\lambda T = 2.9E - 3$. It relates the temperature with the wavelength at maximal intensity. The wavelength at the maximal intensity is approximately $2\mu m$. Plugging this in, one finds that $T = 2.9E - 3/2E - 6 \approx 1500K$, as in choice (D).

Problem 64 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=64*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Radiation Spectra*

Choice-by-choice analysis gives...

(A) The word nuclear sounds questionable, as lines are often due to just spin-splitting.

(B) The wavelengths of absorption spectra are often close to that of emissions spectrum (or overlapping). This is true.

(C) Absorption spectra is often used in astrophysics to determine the contents of stars. Even though mere mostly harmless Earthlings have never visited the sun, the absorption spectra of the sun helps determine its component elements.

(D) Same idea as in the previous choice.

(E) A single atom does not have band spectra. So, yes, band-spectra are due to molecules. (To wit: it only has, say, 2s, 3p, etc., states, and not states that are split in-between—molecules have more degrees of freedom, i.e., they can rotate and vibrate, while atoms can't.)

The remaining choice is (A). Take that.

Problem 65 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=65*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Approximations*

At high temperatures, one has $kT >> h\nu$.

One expands the argument of the denominator according to $e^x \approx 1 + x$ (for small x). The denominator becomes $(1 + h\nu/kT - 1) = h\nu/kT$. Since the whole quantity of the denominator is squared, the top $(h\nu/kT)^2$ term is canceled.

In the numerator, one has $e^{h\nu/kT} \to 1$, since $e^{1/\infty} \to e^0 \to 1$.

Thus, one arrives at choice (D).

One either remembers this fact about solid or one derives it, as shown above. (Also, for low temperatures, at say around 20K for most solids, the Debye T^3 law applies, and specific heat is proportional to T^3. One can't find this result from the Einstein formula, which is why Debye's theory is more accurate for solids.)

Problem 66 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=66$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Radioactivity*

The problem supplies the details for γ-emission and β-emission to be:
$\frac{d\gamma}{dt} = -\frac{ln(1/2)}{24}\gamma$
$\frac{d\beta}{dt} = -\frac{ln(1/2)}{36}\beta$

(Why? Well, one starts with the equation $\frac{dA}{dt} = -kA \Rightarrow lnA/A_0 = -kt \Rightarrow A = A_0 e^{-kt}$, where one integrates both sides. Plugging in the condition for half-life, one has $1/2 = e^{-kT} \Rightarrow k = ln(1/2)/T$, where T is the half-life decay time.)

The total decay is given by $\frac{dR}{dt} = \frac{d\gamma}{dt} + \frac{d\beta}{dt} = -\frac{ln(1/2)}{24}\gamma - \frac{ln(1/2)}{36}\beta = -\frac{ln(1/2)}{\tau}R$.

Thus, $1/24 + 1/36 = 1/T$. Solve for T to get choice (D).

Problem 67 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=67$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Binding Energy*

The binding energy is basically the energy that keeps a nucleus together; and makes its mass slightly different than its constituent particles. (If its mass were exactly the same as its constituent particles, its binding energy would be 0, and it would be unstable.)

The initial binding energy is $U_i = 238(7.6MeV) \approx 240(7.6MeV)$, which is the number of nucleons times the binding energy per nucleon.

The final binding energy is $U_f = 120X + 120X = 240X$, where the daughter nuclei have half the number of nucleons from the given fact that the original nucleus splits into 2 equal fragments.

The difference in binding energy is equal to the kinetic energy. $U_i - U_f = 2 \times -100MeV \Rightarrow 240(7.6MeV - X) = -200MeV$. Solving, one finds that $X \approx 8.5$, as in choice (E).

Problem 68 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=68$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Decay*

Since Lithium has one less electron than Be, one might think this decay is just *beta*-decay. However, *beta*-decay always occurs with a neutrino or anti-neutrino, and since none of the choices show this.

Beta-decay emits either an electron or positron with an antineutrino or neutrino:
${}^Z_A N \rightarrow^Z_{A-1} M + \beta^+ + \nu$
${}^Z_A N \rightarrow^Z_{A+1} M + \beta^- + \bar{\nu}$

(The bit on antineutrinos and neutrinos has to do with conservation of Electron Lepton number L_e. Namely, electrons and neutrinos have $L_e = 1$, while positrons and antineutrinos have $L_e = -1$.)

(Also, α-decay emits a Helium atom with 4 neutrons and 2 protons, so the numbers won't work out here.)

The remaining choice is (E). One can check that it's right by noting that it is the only choice that conserves the electron-lepton number.

Problem 69 $http://grephysics.net/disp.php?yload=prob\&serial=3\&prob=69$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Thin Films*

The incident wavelength changes phase at the air-oil boundary and at the oil-glass boundary. Thus, integer wavelengths produce constructive interference.

The incident wavelength travels through $2t$. Thus, $2t = \lambda \Rightarrow \lambda \approx 240nm$. The closest thickness is 200 nm, so choose choice (B).

Additionally, if one wants a better approximation, one can use the equation $v = c/n = \lambda f \Rightarrow \lambda' = \lambda_0/n$ to determine the wavelength of the beam in oil. Thus, one arrives at the exact answer 200nm. (Addition due to user E123.)

Problem 70 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=70*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Interference*

The single-slit interference equation for bright fringes is given by $m\lambda \propto d\sin\theta \approx d\theta$ (for small angles), where d is the width of the slit and m is an integer.

Since $c = \lambda\nu$, one can relate the above equation to frequency to get $mc/f \propto d\theta$. Increasing frequency would decrease the angle. Thus, the fringes would get closer together. Increasing the frequency by a factor of 2 would decrease the separation by 2, as in choice (B).

Problem 71 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=71*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Doppler Shift*

(The Doppler Shift is derived earlier on in the current author's solutions for GR9677.)

Since the light received has a higher wavelength than the incident wavelength, one concludes that this is a red-shift, that the object is moving away from the Earth.

The Doppler Effect equation gives $\lambda = \sqrt{\frac{1+\beta}{1-\beta}}\lambda_0$, where λ_0 is the wavelength of the source. Thus, $\lambda/\lambda_0 = \sqrt{\frac{1+\beta}{1-\beta}} \approx 6$. Solving, one gets $\beta = 35/37 \Rightarrow v = 35/37c$, which is, by inspection of possible choices, closest to choice (D).

Problem 72 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=72*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Forces*

Before the string is cut, one has the basic static equilibrium condition, for each block $\sum F = m_u a = 0 = T - m_u g - kx$ and $\sum F = m_b a = 0 = kx - m_b g$, where m_u refers to the upper block and m_b refers to the bottom block. Adding, one has $(m_u + m_b)a = 0 = T - m_u g - m_b$.

After the string is cut, the tension goes to 0, but one has a non-zero net acceleration. The top mass has $\sum F = -m_u a = -m_u g - kx$, where the minus sign comes in since the question wants the downward acceleration. The bottom mass has $\sum F = m_b a = kx - m_b g$. But, immediately after the string is cut, the lower mass has 0 acceleration. Thus, $kx = m_b g$. Plugging this into the equation for the upper mass, one finds that $a = 2g$, as in choice (E).

Problem 73 ________________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=73*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Forces*

The net force of the system is $\sum F = (m_a + m_b)a$, and thus the net acceleration due to this force is $a = F/(m_a + m_b)$.

The net force for mass A is just $m_a a = F - F_{ab}$. By Newton's Third Law, $F_{ab} = N$ is just the normal force exerted by A on B.

Solving for the normal force, one finds that $N = F - m_a a = F(1 - m_a/(m_a + m_b)) = F(m_b/(m_a + m_b))$.

Summing up the vertical forces on mass B, and using the fact that the frictional force is just $f = \mu N$, one finds that $\sum F_y = \mu N - m_b g = 0$ for the applied force to balance its mass completely. Thus, $F = (m_a + m_b)g/\mu$, which is approximately 400N, as in choice (D) after plugging in the numbers.

Problem 74 ______ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=74*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Lagrangians*

The Lagrangian equation of motion is given by $\frac{\partial L}{\partial q} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}$ for the generalized coordinate q.

Chunking out the derivatives, one finds that

$\frac{\partial L}{\partial q} = 4bq^3$

$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 2a\ddot{q}$

Setting the two equal to each other as in the Lagrangian equations of motion given above (without undetermined multipliers), one finds that $2bq^3 = a\ddot{q}$, which gives choice (D).

Problem 75 ______ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=75*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Transformations*

Recall the matrix equation for a rotational transformation of a coordinate system,
$R(\theta)$=

$$\cos\theta \qquad \sin\theta \tag{1}$$
$$-\sin\theta \qquad \cos\theta \tag{2}$$

Equating coefficients, one has $\sin\theta = \sqrt{3}/2$. Thus, $\theta = \pi/3$, which corresponds to a 60-degree counter-clockwise rotation, as in choice (E). (Recall the unit circle.)

Problem 76 ______ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=76*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Solid State Physics*

Elimination time:

(A) Electrons have less degrees of freedom than free atoms, since electrons are bound to potential wells.

(B) Why not?

(C) The electrons do indeed form a degenerate Fermi gas, since the ratio of the fraction of electrons in the ground-state is given by $N_0/N = 1 - (T/T_F)^{3/2}$, where $T_F \approx 30,000$ K for most metals.

(D) The electrons in metal travel at a drift velocity of about $10^{-6} m/s$. Not quite fast enough to be relativistic.

(E) Electron interaction with phonons has nothing to do with their mean kinetic energy.

Problem 77 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=77*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Maxwell-Boltzmann Distributions*

The Maxwell-Boltzmann distribution is $N \propto ge^{-\epsilon/(kT)}$, where g is the degeneracy.

Given $\epsilon_a = 0.1+\epsilon_b$, one finds the ratio of distributions (thus ratio of numbers) to be $e^{-(0.1+\epsilon_b)/kT}/e^{-\epsilon_b/kT} = e^{-0.1/kT}$.

The problem gives $kT = 0.025$ eV, and thus the above ratio becomes $e^{-.1/.025} = e^{-4}$, as in choice (E).

Problem 78 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=78*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Particle Physics*

A few lines from the Particle Physics chapter of my upcoming book (title tentative, to be publicized on this website eventually), *Reviewing Forgotten Physics for the GRE, Prelim's and Qualifier's*:

Baryons and mesons are hadrons. (Hadrons are particles that interact with the strong nuclear force.) Baryons have a baryon number of $B = \pm 1$, while mesons have $B = 0$. Nucleons (which are baryons) have $B = 1$, while antinucleons (antineutrons and antiprotons) have $B = -1$.

Leptons (electrons, neutrinos, muons, and tau's) interact with the weak nuclear force. There are three kinds of lepton numbers. There is the electron lepton number L_e and there is a muon lepton number, as well as a neutrino lepton number.

Electrons β^- and the electron neutrino ν_e have $L_e = 1$, while positrons (antielectrons) β^+ and the electron antineutrino $\bar{\nu}_e$ have $L_e = -1$. All other particles have $L_e = 0$

Conservation of baryon number or lepton number is just summing up either the baryon number or the lepton number on both sides of the reaction.

Conservation of the numbers above explain for why a reaction like $\mu \rightarrow \beta^- + \nu_\mu + \bar{\nu}_e$ must occur in lieu of $\mu \rightarrow \beta^- + \bar{\nu}_e$ or $\mu \rightarrow \beta^- + \nu_e$ or $\mu \rightarrow \beta^- + \bar{\nu}_\mu$ or $\mu \rightarrow \beta^- + \nu_\mu$.

For the first reaction, the electron Lepton numbers are $0 \rightarrow 1 + 0 - 1$, and thus L_e is conserved. (Muon Lepton numbers are $1 \rightarrow 0 + 1 + 0$)

For the second reaction, the muon lepton numbers are $1 \rightarrow 0 + 0$, but that does not add up, and so L_μ is not conserved.

For the third reaction, the electron Lepton numbers are $0 \rightarrow 1 + 1$, but that does not add up, and so L_e is not conserved.

As an exercise, one can calculate the fourth and fifth reaction.

(If you're beginning to feel like this is more alchemy than physics, then note that the Standard Model of particle physics is just a transitory theory, like alchemy was to chemistry. It is to be replaced soon with a less ad-hoc theory, perhaps by *you*.)

With respect to this question on the GRE exam, one sees that lepton number would not be conserved for any of the possible cases of the latter reactions.

Problem 79 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=79*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Rest Mass*

The total relativistic energy is $10GeV = \gamma mc^2 = \sqrt{p^2c^2 + m^2c^4}$. The total relativistic momentum is $p = 8GeV/c = \gamma mv$.

Plugging the momentum into the first equation, one has $(8^2 + m^2c^2)c^2 = 10^2 \Rightarrow m^2c^4 = 100 - 64 = 36 \Rightarrow m = 6GeV/c^2$, as in choice (D).

Problem 80 http://grephysics.net/disp.php?yload=prob&serial=3&prob=80
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type
Special Relativity → *Addition of Velocities*

If one forgets the addition of velocity formula, one can quickly derive it from the Lorentz transformation (which one really ought to remember),
$x = \gamma(x' + vt')$
$t = \gamma(t' + vx'/c^2)$
where the unprimed system is the rest frame.
One finds that $u_x = dx/dt = \frac{d}{dt}\gamma(x' + vt') = \gamma \frac{d}{dt'}(x' + vt')\frac{dt'}{dt} = \gamma(u_x^{'} + v)$.
From the time transformation, one gets $\frac{dt}{dt'} = \gamma(1 + vu_x^{'}/c^2)$, where $u_x^{'}$ is the velocity of the particle moving in the primed coordinate system. Plugging the inverse of that into the velocity in the unprimed system, one gets
$u_x = \frac{u_x^{'}+v}{1+vu_x^{'}/c^2}$.
The problem supplies the following values: $v = c/2$, $u_x^{'} = c/n = 3c/4$ (since $n = 4/3$). Plugging them in the u_x equation, one finds that $u_x = \frac{3c/4+c/2}{1+3/8} = 10c/11$, as in choice (D).

Problem 81 http://grephysics.net/disp.php?yload=prob&serial=3&prob=81
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type
Quantum Mechanics → *Angular Momentum*

Recall the angular momentum eigen-equations, $L^2\psi = \hbar^2 l(l+1)\psi$ and $L_z\psi = m\hbar\psi$.
The problem wants $L^2\psi = 6\hbar^2\psi$ and $L_z\psi = -\hbar\psi$. Matching coefficients with the above equations, one finds that
$l(l+1) = 6$ and $m = -1$. Solving, one finds that $l = 2, -3$. Any one of the spherical harmonics with Y_2^{-1} or Y_{-3}^{-1} would work. So, since the second spherical harmonic isn't listed, take choice (B).

Problem 82 http://grephysics.net/disp.php?yload=prob&serial=3&prob=82
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type
Quantum Mechanics → *Addition of Angular Momentum*

For 2 electrons, one recalls that the there are three spin triplet states and one spin singlet state (hence their names). One can apply the lowering operator multiple times to the up-up state to arrive at, respectively, the up-down state, the down-up state, and then the down-down state (the spin-singlet state).
Applying the lowering operator to choice I, one gets choice III. Applying the lowering operator to that, one gets down-down. These are the 3 spin-triplet states.
By orthogonality, the down-down state has a negative sign. So, only choices I and III are in the triplet configuration.
(David J. Griffiths vanity alert—the QM problems thus far are all straight out of his textbook, *An Introduction to Quantum Mechanics*.)

Problem 83 http://grephysics.net/disp.php?yload=prob&serial=3&prob=83
For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type
Quantum Mechanics → *Basis*

One can chunk out the 2-by-2 matrices by choosing a basis for z. Choose the easiest, $|\uparrow\rangle = (1,0)$ and $|\downarrow\rangle = (0,1)$.

One can tell by inspection that choice (A) can't be it, nor can choices (D) and (E) (since there aren't any imaginary numbers involved for x).

Recall that $S_i = \hbar/2\sigma_i$.

Plug in the deductions above into choice (B) to get $\psi_b = (|\uparrow\rangle + |\downarrow\rangle)/\sqrt{2} = (1,1)/\sqrt{2}$. Multiply that with the Pauli matrix to get $S_x\psi_b = \hbar/2\psi_b$.

Plug in the deductions above into choice (C) to get $\psi_c = (|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2} = (1,-1)/\sqrt{2}$. Multiply that with the Pauli matrix to get $S_x\psi_c = -\hbar/2\psi_b$.

Problem 84 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=84*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Selection Rules*

The selection rules are $\Delta l = \pm 1$ and $\Delta m = 0, \pm 1$.

The transitions have the following quantum numbers:

A: $\Delta l = 0$ and $\Delta j = 0$

B: $\Delta l = -1$ and $\Delta j = -1$

C: $\Delta l = -1$ and $\Delta j = 0$

By the selection rules above, transition A is forbidden in Δl. Transitions B and C both work out, since they do not violate the Δl rule.

Take choice (D).

Problem 85 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=85*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Resistivity*

This is a baby circuits problem once one calculates the net resistance.

Recall that resistance is related to length and area by $R = \rho l/a$.

Thus, the resistance of the thin and long wire is $R_{tl} = \rho 2L/A$, while the resistance of the fat and short wire is $R_{fs} = \rho L/(2A)$.

The resistors are in series, and so the equivalent resistance is $R_{eq} = \rho L/A(2 + 1/2) = 5\rho L/(2A)$.

Plug that into Ohm's Law to find the current $\Delta V = IR_{tot} = 8 - 1 = 7$. Thus, $I = 14A/(5\rho L)$.

To find the potential at the junction, use Ohm's Law again to get $V_{junction} = 1 + IR_{fs}$. Chunking out the algebraic expressions, one gets 2.4 V, as in choice (A0.

Problem 86 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=86*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Faraday Law*

The voltage induced is equal to the change in magnetic flux $\epsilon = -\frac{\partial \Phi}{\partial t}$, where $\Phi = \int \vec{B} \cdot d\vec{A}$.

Noting the initial condition ($\Phi(t = 0) = 0$), since the field and area normal are perpendicular), one finds that $B \cdot dA = B \sin(\omega t)\pi r^2$. Thus, $d\Phi/dt = \omega B \cos(\omega t)\pi r^2$.

Now, to find the current, one uses Ohm's Law in Faraday's Law to get $IR/N = \dot{\Phi}$, where N is the number of turns. Thus, $I = N/R\dot{\Phi} = NB\omega/R \cos(\omega t)\pi r^2 = 15/2 \times 300/9 \cos(\omega t)\pi(1/100)^2 = 250E - 4\cos(\omega t)$. This is choice (E).

Problem 87 ______________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=87*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Potential*

Since the potential is constant inside the sphere, one has the field inside to be 0—even though it is not a conductor.

Thus, the force contribution comes from the other sphere. By, Gauss' Law, the other sphere acts just like a point-charge, and thus its field contribution is $E = \frac{Q}{4\pi\epsilon_0(10d-d/2)^2} = \frac{4Q}{4\pi\epsilon_0 361(d)^2}$.

The force on the test charge is thus $F = qE = \frac{Q}{\pi\epsilon_0 361(d)^2}$, which is just choice (A).

Problem 88 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=88*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Magnetism → *Biot-Savert Law*

The Biot-Savart law is $d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\hat{r}}{r^2}$.

One notes that the non-bent wires have $dl||r$, and thus their cross-products go to 0.

The only contribution to the field is from the curved segment. Its field is $B = \frac{\mu_0 I}{4\pi}\frac{\theta R}{R^2}$, where $dl = \theta R$ is just the arclength. Simplifying, one gets choice (C).

Problem 89 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=89*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Angular Momentum*

The initial angular momentum is $L_0 = (MR^2/2 + mR^2)\omega = (M/2 + m)R^2\omega$

The final angular momentum is $L_f = (MR^2/2)\omega^{'}$, since the radius of gyration of the young padawan is now 0.

Conservation of angular momentum requires that $L_0 = L_f \Rightarrow \omega^{'} = (M/2+m)/(M/2) = 2.8 rad/s$.

Problem 90 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=90*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Normal Modes*

Note that $\omega = \sqrt{\frac{k_{eff}}{m_{eff}}}$:

For figure 1, the potential energy is $U = 2\times 1/2kx^2 = kx^2$. The kinetic energy is just $T = 1/2m\dot{x}^2$. Thus, $k_{eff} = 2k$ and $m_{eff} = m$. Thus, $\omega_1 = \sqrt{2k/m}$.

For figure 2, the potential energy is $U = 2\times 1/2k(x/2)^2$, since each spring travels only half as far. The kinetic energy is the same as in figure 1. Thus, $k_{eff} = k/2$ and $m_{eff} = m$. So, one has, $\omega_2 = \sqrt{k/2m}$

Since $\omega = 2\pi f = 2\pi/T$, the period $T_1/T_2 = \omega_2/\omega_1 = \sqrt{(1/2)/(2)} = 1/2$, as in choice (A).

Problem 91 ——— *http://grephysics.net/disp.php?yload=prob&serial=3&prob=91*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Energy*

Conservation of energy gives $MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$. $v = R\omega$, and thus the equation becomes $MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I(v/R)^2$.

Thus, $I = 2(MgH - Mv^2)R^2/v^2$. (Note that the height of the center of mass is the same at both the end and the start, thus the extra bit of the potential energy MgR cancels out. Thanks to the user keflavich for this correction.)

Given $v^2 = 8gH/7$, one plugs it into the equation above to get $I = 2(MgH - M4gH/7)R^2/(8gH/7) = 2(3MgH/7)R^2/(8gH/7) = 3MR^2/4$, as in choice (B).

Problem 92 _____________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=92*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Hamiltonian*

The Hamiltonian is just the sum of the kinetic and potential energy, $H = T + V$.

The kinetic energy due to each mass is $T_i = p_i^2/(2m)$. The potential energy is just $U = 1/2k(\Delta l)^2$. $\Delta l = l - l_0$, and thus factoring out the 1/2, one arrives at choice (E).

Problem 93 _____________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=93*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Most probable value*

Recall the definition of probability, $P = |\psi|^2 dV = |\psi|^2 4\pi r^2 dV$. (The radial probability distribution P_r is related to the probability P by $P = \int P_r dr$.) The most probable value is given by the maximum of the probability. Taking the derivative with respect to r, one has this condition for a maximum (the second derivative shows that it's concave up)

$dP_r/dr = 4\pi r^2 d|\psi|^2/dr + 8\pi r|\psi|^2 = 0$.

The problem gives $\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$. Thus, $|\psi|^2 = \frac{1}{\pi a_0^3} e^{-2r/a_0}$ and $d|\psi|^2/dr = \frac{-2}{\pi a_0^3 a_0} e^{-2r/a_0}$.

Plug this into the expression for dP/dr, solve for r to find that the most probable distance is just the Bohr radius, as in choice (C).

Problem 94 _____________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=94*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Perturbation Theory*

The energy for first-order perturbation theory of $H = H_0 + \Delta H$ (H_0 is the known Hamiltonian and ΔH is the perturbed Hamiltonian) is given by $E_1 = \langle \psi_0 | \Delta H \psi \rangle_0$, where the wave-functions are the unperturbed ones.

Thus, the problem amounts to calculating $E_1 = \langle n | V(a + a^\dagger)^2 | n \rangle$. This is just raising and lowering operator mechanics.

$(a + a^\dagger)^2 = a^2 + a^{\dagger 2} + aa^\dagger + a^\dagger a$. But, after bra-ketting, one finds that the expectation value of a^2 and $a^{\dagger 2}$ are 0, since n and $n+2$, $n-2$ are orthogonal. Thus, the problem becomes,

$E_1 = \langle n | (aa^\dagger + a^\dagger a) | n \rangle$. Applying the given eigen-equations, one finds that $E_1 = \langle n | (aa^\dagger + a^\dagger a) | n \rangle = (2n+1)V$. For $n = 2$, one finds $E_1 = 5V$, as in choice (E).

(Note that: $a^\dagger a|n\rangle = a^\dagger \sqrt{n}|n-1\rangle = n|n\rangle$ and $aa^\dagger|n\rangle = a\sqrt{n+1}|n+1\rangle = (n+1)|n\rangle$ and $a^2 = a\sqrt{n}|n-1\rangle = \sqrt{n}\sqrt{n-1}|n-2\rangle$.)

Problem 95 _____________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=95*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Dielectric*

No real calculations involved here. Since $E \propto 1/\epsilon$, if one adds in a dielectric then $\epsilon = K\epsilon_0$. Since $E_0 \propto 1/\epsilon_0$, this means that $E^{'} = E_0/K$.

Problem 96

http://grephysics.net/disp.php?yload=prob&serial=3&prob=96

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Symmetry*

By symmetry, the *total* power radiated is 0. It expands inwards and outwards at the same frequency.

Problem 97

http://grephysics.net/disp.php?yload=prob&serial=3&prob=97

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Refraction*

From Snell's Law, one obtains $\sin\theta = n(\lambda)\sin\theta^{'}$, since the index of refraction of air is about 1. Now, differentiate both sides with respect to λ.

$$\frac{d}{d\lambda}\sin\theta = \frac{d}{d\lambda}(n(\lambda)\sin\theta^{'}) \tag{3}$$

$$0 = \frac{dn(\lambda)}{d\lambda}\sin\theta^{'} + (n(\lambda)\cos\theta^{'})\frac{d\theta^{'}}{d\lambda} \tag{4}$$

$$\delta\theta^{'} = |\tan\theta^{'}/n\frac{dn(\lambda)}{d\lambda}\delta\lambda|, \tag{5}$$

which gives choice (E) to be the angular spread.

This solution is due to ShyamSunder Regunathan.

Problem 98

http://grephysics.net/disp.php?yload=prob&serial=3&prob=98

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Average Energy*

The average energy is given by $\sum_i E_i e^{-E_i/kT}/Z$, where $Z = \sum_i e^{-E_i/kT}$ is the partition function.

The above is just the stat mech generalization of the baby-averaging $\bar{u} = \sum_i U_i/N$, where N is the number of energies summed over. Z is like a generalized N.

Problem 99

http://grephysics.net/disp.php?yload=prob&serial=3&prob=99

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Conservation of Energy*

The tricky part of this problem is to note that the momentum of the photon is shared *equally* between all three final particles. (This insight was supplied by Felipe Birk). Thus, $p_e = p/3$, where p is the momentum of the photon and p_e is the momentum of the *final* electron or photon.

The initial energy *before* the photon strikes the electron is $E_0 = pc + m_e c^2$, which is just the energy of the photon plus the rest energy of the electron.

The final energy after the collision is $E_f = 3\sqrt{(pc/3)^2 + (m_ec^2)}$, which is the sum total energy of all three final particles, i.e., the positron and two electrons. (A positron is the electron's antiparticle, and thus they have the same mass.) Note that the momentum split relation mentioned above is used to equate the final particle momentum with the initial photon momentum.

Conjure up the good conservation of energy idea. Equating $E_0 = E_f$, one gets $(pc)^2 + (m_ec^2)^2 + 2pcm_ec^2 = 9((pc/3)^2 + (m_ec^2)^2)$. Canceling the $(pc)^2$ terms on both side, then solving, one arrives at $pc = 4m_ec^2$, which is choice (D).

Problem 100 ________ *http://grephysics.net/disp.php?yload=prob&serial=3&prob=100*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Interferometer*

This is a nice problem.

An interferometer, like its name suggests, has to do with interference. Namely, it splits light-beams through the beam-splitters (BS) half and half, then recombines them at the end—the end result shows interference which varies depending on whether the moveable mirror is placed a quarter of a wavelength away (so that the path difference, which is twice that, is half a wavelength).

A fringeshift occurs whenever $d = \lambda$. Thus $m = 2d/\lambda$, where m is the number of fringes and λ is the wavelength.

Using the information for the red beam, one can find $d = m\lambda/2 = 85865 \times 632.82/2$. Applying the same equation for the green beam, one finds that $\lambda = 2d/m = 85865 \times 632.82/100000 \approx 540nm$, as in choice (B).

